

$$\text{Then } y_p'' = u_1' y_1' + u_2' y_2' + u_1 y_1'' + u_2 y_2''$$

Substituting y_p, y_p' and y_p'' into (1) and using the fact that y_1 & y_2 are solutions of the homogeneous part, we get

$$u_1' y_1' + u_2' y_2' = g(x). \quad \text{--- (5)}$$

We solve for u_1' & u_2' from (4) & (5) using Cramer's Rule to get:

$$u_1' = \frac{-g(x)y_2(x)}{W(y_1, y_2)}$$

$$u_1'(x) = \frac{-y_2 g(x)}{W(y_1, y_2)} \quad \text{--- (6)}$$

$$u_2'(x) = \frac{y_1 g(x)}{W(y_1, y_2)}$$

$$\text{where } W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \quad \text{--- (7)}$$

is the Wronskian of the fundamental solutions y_1 & y_2 .

Integrating (6) we find:

$$\left. \begin{aligned} u_1(x) &= - \int \frac{y_2 g(x)}{W(y_1, y_2)} dx + C_1 \\ u_2(x) &= \int \frac{y_1 g(x)}{W(y_1, y_2)} + C_2 \end{aligned} \right\} \quad \text{--- (8)}$$