

	Fishing	Carpentry
Bicycle (X_1)	2	1
Car (X_2)	1	1
Total	100	80

Therefore constraint 1 may be expressed by:

$$2x_1 + x_2 \leq 100 \quad (2)$$

and constraint 2 may be written as

$$x_1 + x_2 \leq 80 \quad (3)$$

Finally, we express the fact that at most 40 bicycles per week can be sold by limiting the weekly productions of bicycles to at most 40 bicycles. This yields the following:

$$x_1 \leq 40 \quad (4)$$

Sign Restriction: To complete the formulation of a linear programming problem, the following questions must be answered for each decision variable. Can the decision variable only assume nonnegative values, or is the decision variable allowed to assume both positive and negative values?

If a decision variable X_1 can only assume nonnegative values, we add the sign restriction $X_1 \geq 0$. If a variable X_2 can assume both positive and negative (or zero) values, we say that X_1 is unrestricted in sign (often abbreviated urs). For Mapulanga problem, it is clear that

$$x_1 \geq 0 \text{ and } x_2 \geq 0$$

Combining the sign restriction $X_1 \geq 0$ and $X_2 \geq 0$ with the objective function (1) and constraints (2) – (4) yields the following optimization model:

$$\text{Max } Z = 15\,000x_1 + 10\,000x_2 \text{ subject to (s.t)} \quad (1)$$

$$x_1 + x_2 \leq 100 \quad (\text{Finishing constraint}) \quad (2)$$

$$x_1 + x_2 \leq 80 \quad (\text{Carpentry constraint}) \quad (3)$$

Feasible Region and Optimal Solution

Definition: The **feasible region** for an LP is the set of all points satisfying all the LP's constraints and all the LP's sign restrictions. Any other point that is not in an LP's feasible region is said to be an **infeasible point**.

Definition: For a maximization problem, an optimal solution to an LP is a point in the feasible region with the largest objective function value. Similarly, for a minimization problem, an optimal solution is a point in the feasible region with the smallest objective function value.

Example (2) Dorian Auto Manufacturers luxury cars and trucks. The Company believes that its most likely customers are high-income women and men. To reach these groups, Dorian Auto has embarked on an ambitious TV advertising campaign and has decided to purchase 1 minute Commercial Spots on two types of programs. Comedy shows and football games. Each comedy Commercial is seen by 7 million high-income women and 2 million high-income men. Each football commercial is seen by 2 million high-income women and 1 million high-income men. A 1 minute comedy ad costs K50,000, and a 1 minute football ad costs K100,000. Dorian would like the commercials to be seen by at least 28 million high-income women and 24 million high income men. Use linear programming to determine how Dorian Auto can meet its advertising requirements at minimum cost.

Example (3) An auto Company manufactures cars and trucks. Each vehicle must be processed in the paint shop and body assembly shop. If the paint shop were only painting trucks, 40 per day could be painted. If the paint shop were only painting cars, 60 per day could be painted. If the body shop were producing cars, it could process 50 per day. If the body shop were only producing trucks, it could process 50 per day. Each truck contributes K1500 000 to profit, and each car contributes K100 000 to profit. Use linear programming to determine a daily production schedule that will maximize the company's profits.