5) If $f(x) = 2x^2 - 8x + 10$

i) ***Express*** $f(x)$ is the form $a(x+b)^2 + c$, where a, b and c are constant

ii) hence, ***state*** the coordinates of the stationary point of $f(x)$ and state its type.

Solution

i) $2x^2 - 8x + 10 \equiv a\,(x+b)^2 + c$

$$\equiv a\,(x^2 + 2bx + b^2) + c$$

$$\equiv ax^2 + 2abx + ab^2 + c$$

$a = 2$ $\quad 2ab = -8$ $\quad ab^2 + c = 10$

$4b = -8$ $\quad 2(-2)^2 + c = 10$

$b = -2$ $\quad 8 + c = 10$ $\quad c = 2$

$$f(x) = 2(x-2)^2 + 2$$

ii) Stationary point (2, 2) type of stationary point is **minimum**

---

6) ***put*** $x^2 + 4x + 3$ in the form $(x+a)^2 + b$ stating the values of a and b.

Solution

$x^2 + 4x + 3 \equiv (x+2)^2 - 4 + 3$

$x^2 + 4x + 3 \equiv (x+2)^2 - 1,$ $\quad a = 2\,, b = -1$

---

7) ***Put*** $2 + 2x - x^2$ in the form $h - (x+p)^2$ stating the values of h and p.

Solution

$-x^2 + 2x + 2 = -1(x^2 - 2x - 2)$

$$= -1[(x-1)^2 - 1 - 2] = -1[(x-1)^2 - 3]$$

$$= -(x-1)^2 + 3 = 3 - (x-1)^2 \qquad h = 3 \text{ and } p = -1$$

12) *Express* $8x - x^2$ in the form $a - (x + b)^2$, stating the values of a and b

Solution

$$-x^2 + 8x = -1(x^2 - 8x) = -1[(x - 4)^2 - 16]$$

$$= 16 - (x - 4)^2 \qquad a = 16, \ b = -4$$

✍ ***Test yourself:***

1) Given that $x^2 - 4x + 7 \equiv (x-a)^2 + b$. ***Find*** the value of a and b.

---

2) ***Express*** each of the following in the form of $(x+a)^2 + b$; stating the value of a and b. Hence, ***find*** the coordinates of the vertex and ***state*** whether it is minimum or maximum.

a) $x^2 + 2x + 2$

---

b) $x^2 - 8x - 3$

Quadratic Inequalities

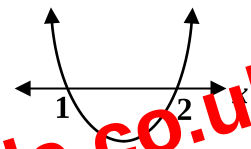
How to solve quadratic inequality:

- Change the inequality to an equation.
- Solve the equation (find the roots).
- Sketch the curve and locate the roots.
- State the range of values of *x* satisfying the inequality.

Examples:

1) ***Solve*** each of the following inequality;

a) $x^2 - 3x + 2 \geq 0$



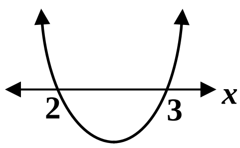
Solution

$x^2 - 3x + 2 = 0 \Rightarrow (x-2)(x-1) = 0 \Rightarrow x = 2 \quad , \quad x = 1$

$\therefore x \leq 1 \quad \text{and} \quad x \geq 2$

---

b) $x^2 - 5x + 6 \leq 0$



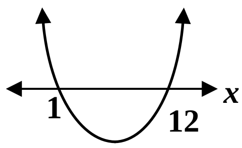
Solution

$x^2 - 5x + 6 = 0 \Rightarrow (x-2)(x-3) = 0 \Rightarrow x = 2 \quad , \quad x = 3$

$\therefore 2 \leq x \leq 3$

c) $x^2 + 12 < 13x$

Solution



$x^2 - 13x + 12 < 0$

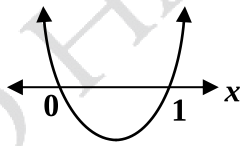
$x^2 - 13x + 12 = 0 \Rightarrow (x-1)(x-12) = 0 \Rightarrow x = 1 \quad , \quad x = 12$

$\therefore 1 < x < 12$

---

d) $x^2 > x$

Solution



$x^2 - x > 0$

$x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0 \quad , \quad x = 1$

$\therefore x < 0 \quad$ and $\quad x > 1$

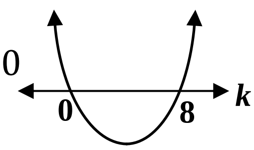
---

2) *Find* the range of values of k for which the equation $kx^2 + kx + 2 = 0$ has no real roots.

Solution

$a = k \quad , \quad b = k \quad , \quad c = 2$

$\Delta = b^2 - 4ac < 0 \Rightarrow k^2 - 4(k)(2) < 0 \Rightarrow k^2 - 8k < 0$



$k^2 - 8k = 0 \Rightarrow k(k-8) = 0 \Rightarrow k = 0 \quad , \quad k = 8$

$\therefore 0 < k < 8$

✍ ***Test yourself:***

1) ***solve*** $x + 2y = 5$, $x^2 + y^2 = 10$

---

2) A line has the equation $x + 3y = 1$ and a curve has equation $x^2 + 3x + 5y = 20$. ***Find*** the coordinates of the points of intersection.

---

3) Given that the line $y = 2x + c$touches the curve $y = x^2$, ***find*** the value of c.

14) The equation $x^2 + px + q = 0$, where p and q are constants, has roots -3 and 5

i) ***Find*** the values of p and q.

---

ii) Using these values of p and q, ***find*** the value of the constant r for which the equation $x^2 + px + q + r = 0$ has equal roots.

18) A line has equation $y = kx + 6$ and a curve has equation $x^2 + 3x + 2k$, where k is a constant.

i) For the case where k = 2, the line and the curve intersect at points A and B. ***Find*** the distance AB and the coordinates of the mid-point of AB.

ii) ***Find*** the two values of k for which the line is a tangent to the curve.

35) If $y = 7 - 10x - x^2$

i) ***Express*** $7 - 10x - x^2$ in the form $a - (x - b)^2$

ii) ***State*** the max/min value of y and the value of x at which it occurs

iii) ***Write down*** the equation of the line of symmetry of the curve $y = 7 - 10x - x^2$

ii) $y^2 + 2x = 13 \rightarrow (1)$ $\quad 2y + x = k \rightarrow (2)$

$$from\ (2): \quad x = k - 2y$$

$$into(1): \quad y^2 + 2(k - 2y) = 13$$

$$y^2 + 2k - 4y - 13 = 0$$

$$\therefore y^2 - 4y + 2k - 13 = 0$$

For the line to be a tangent to the curve $\Delta = 0$

$a = 1 \quad b = -4 \quad c = 2k - 13$

$$\therefore b^2 - 4ac = 0$$

$$16 - 4(1)(2k - 13) = 0$$

$$16 - 8k + 52 = 0$$

$$68 = 8k \rightarrow k = \frac{68}{8} = 8.5$$

---

13) i) $y = 2x^2 - 8x + 9 \rightarrow (1)$, $x + y = 3 \rightarrow (2)$

from (1) into (2):

$$x + 2x^2 - 8x + 9 = 3$$

$$2x^2 - 7x + 6 = 0$$

$$x = \frac{3}{2} \qquad x = 2$$

---

ii) For the curve C, $y = 2x^2 - 8x + 9$ we will get the x-coordinate of the stationary point (vertex)

$$x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = 2\ ,$$

Which is one of the points in part (i)

14) i) ***Method (1):***

$x = -3$ & $x = 5$ are roots of the equation, the equation could be written in the form

$(x+3)(x-5) = 0$

$x^2 - 2x - 15 = 0 \leftrightarrow x^2 + px + q = 0$

$\therefore p = -2 \quad , \quad q = -15$

Method (2): (Longer Method)

$x^2 + px + q = 0$

At $x = -3$ $\quad (-3)^2 + p(-3) + q = 0 \qquad q - 3p + q = 0$

$\therefore -3p + q = -q \quad \rightarrow (1)$

At $x = 5$ $\quad (5)^2 + p(5) + q = 0$

$25 + 5p + q = 0 \qquad 5p + q = -25 \quad \rightarrow (2)$

Now solve (1) & (2)simultaneously:

$-3p + q = -q$

$\underline{-5p - q = 25}$

$-8p = 16 \qquad p = -2 \quad , \quad q = -15$

---

ii) $x^2 + px + q + r = 0$

$x^2 - 2x - 15 + r = 0$

$a = 1 \qquad b = -2 \qquad c = r - 15$

For the equation to have equal roots $\Delta = 0$

$b^2 - 4ac = 0$

$(-2)^2 - 4(1)(r - 15) = 0$

$4 - 4r + 60 = 0$

$64 = 4r \quad \rightarrow \quad r = 16$

<u>At m = 2 into equation (3):</u>

$x^2 - 6x + 9 = 0$

$x = 3 \quad , \quad y = 2$

∴Coordinates of point of tangency is $(3, 2)$

---

iii) $x^2 - 4x + 5 \equiv (x + a)^2 + b$

$\equiv x^2 + 2ax + a^2 + b$

$\therefore\ 2a = -4 \qquad a^2 + b = 5$

$a = -2 \qquad 4 + b = 5$

$b = 1$

$\therefore\ x^2 - 4x + 5 \equiv (x - 2)^2 + 1$

∴ Coordinates of minimum point on the curve $y = x^2 - 4x + 5\ is\ (2, 1)$

---

24) i) $y = 2x \quad \to (1) \qquad y = 2x^5 + 3x^3 \quad \to (2)$

<u>From (2) into (1):</u>

$2x^5 + 3x^3 = 2x$

$2x^5 + 3x^3 - 2x = 0$

$x(2x^4 + 3x^2 - 2) = 0$

$\therefore\ Either\ \ x = 0\ \ (rejected)\ \ or\ \ 2x^4 + 3x^2 - 2 = 0$ Equation satisfied by A & B

---

ii) $2x^4 + 3x^2 - 2 = 0$

<u>let m $=x^2$</u> $\qquad \therefore\ 2m^2 + 3m - 2 = 0$

$m = \frac{1}{2} \qquad\qquad m = -2$

$x^2 = \frac{1}{2} \qquad\qquad x^2 = -2$ (rejected)

$x_B = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \quad , \quad x_A = -\sqrt{\frac{1}{x}} = -\frac{\sqrt{2}}{2}$

$y_{B=\sqrt{2}} \quad , \quad y_{A=-\sqrt{2}} \qquad \therefore\ A = \left(\frac{-\sqrt{2}}{2}, -\sqrt{2}\right) \quad , \quad B = \left(\frac{\sqrt{2}}{2}, \sqrt{2}\right)$