Q.4 $y = \log x$

diff w.r.t. x

$\frac{dy}{dx} = \frac{1}{x}$

diff w.r.t. x

$\frac{d^2y}{dx^2} = \frac{-1}{x^2}$

Q.5 $y = x^3 \log x$

diff w.r.t. x

$\frac{dy}{dx} = x^3 \cdot \frac{1}{x} + \log x \cdot 3x^2$

$= x^2 + 3x^2 \log x$

$y_1 = x^2 (1 + 3 \log x)$

diff w.r.t x

$\frac{d^2y}{dx^2} = x^2 \left\{\frac{3}{x}\right\} + \ldots \cdot \{2x\}$

$= 3x + 6x \log x$

$= 5x + 6x \log x$

Q.6 $y = e^x \cdot \sin 5x$

diff w.r.t x

$\frac{dy}{dx} = e^x \{(\cos 5x) \cdot 5\} + \sin 5x \{e^x\}$

$= e^x \{5 \cdot \cos 5x + \sin 5x\}$

diff w.r.t x

$\frac{d^2y}{dx^2} = e^x \cdot \{25 \cdot (-\sin 5x) 5 + 5 \cos 5x\} + \{5 \cos 5x + \sin 5x\} e^x$

$= e^x \{-25 \sin 5x + 5 \cos 5x + 5 \cos 5x + \sin 5x\}$

$$(1-x^2)\frac{d^2y}{dx^2} = x\cdot\frac{dy}{dx}$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$

OR

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

Square both sides

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{(1-x^2)}$$

$$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 1$$

diff w.r.t. x

$$(1-x^2)\,2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2(-2x) = 0.$$

$$2\frac{dy}{dx}\left[(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}\right] = 0$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0.$$

Q14 If $y = (\tan^{-1}x)^2$ Show that $(x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$

→ Given $y = (\tan^{-1}x)^2$

diff w.r.t. x

$$y_1 = 2(\tan^{-1}x)\cdot\frac{1}{1+x^2}$$

Squaring both sides

$$(y_1)^2 = 4(\tan^{-1}x)^2\cdot\frac{1}{(1+x^2)^2}$$

$$y_1^2 = \frac{4y}{(1+x^2)^2}$$

$$(1+x^2)^2\cdot y_1^2 = 4y$$

diff w.r.t. x.

$$(1+x^2)\cdot 2y_1\cdot y_2 + y_1^2(1+x^2)(2x) = 4y_1.$$