

Derivative of one function w.r.t another function

Let u & v be 2 functions of x , then derivative of u w.r.t. v can be obtained by

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}}$$

Problems

1) Diff $\sin^2 x$ w.r.t $e^{\cos x}$:

→ Let $u = \sin^2 x$ & $v = e^{\cos x}$.
Consider $u = \sin^2 x$

diff w.r.t. x

$$\frac{d}{dx} \cdot \frac{du}{dx} = 2 \sin x \cdot \cos x$$

Consider $v = e^{\cos x}$

diff w.r.t. x

$$\frac{dv}{dx} = e^{\cos x} (-\sin x)$$

$$\frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2 \sin x \cdot \cos x}{e^{\cos x} \cdot (-\sin x)}$$
$$\frac{d \sin^2 x}{d e^{\cos x}} = - \frac{2 \cos x}{e^{\cos x}}$$

2) Diff $\tan^{-1} x$ w.r.t $\log x$

→ Let $u = \tan^{-1} x$ & $v = \log x$
Consider $u = \tan^{-1} x$

diff w.r.t. x

$$\frac{du}{dx} = \frac{1}{1+x^2}$$

Consider $v = \log x$

diff w.r.t. x

$$\frac{dv}{dx} = \frac{1}{x}$$