More exercises for you to try

1 a) Write down the derivatives of each of:

$$x^3, \qquad x^3 + 17, \qquad x^3 - 21$$

b) Deduce that $\int 3x^2 \mathrm{d}x = x^3 + c$.

2. What is meant by the term 'integrand'?

3. Explain why, when finding an indefinite integral, a constant of integration is always needed.

Answer

2. A Table of Integrals

We could use a table of derivatives to find integrals, but the more common ones are usually found in a 'Table of Integrals' such as that shown below. You could check the entries in this table using your knowledge of differentiation. Try this for yourself.

Table of integrals

function $f(x)$	indefinite integral $\int f(x)\mathrm{d}x$
constant, k	$kx + c$
x	$\frac{1}{2}x^2 + c$
x^2	$\frac{1}{3}x^3 + c$
x^n	$\dfrac{x^{n+1}}{n+1} + c, \quad n \neq -1$
x^{-1} (or $\dfrac{1}{x}$)	$\ln\|x\| + c$
$\cos x$	$\sin x + c$
$\sin x$	$-\cos x + c$
$\cos kx$	$\dfrac{1}{k}\sin kx + c$
$\sin kx$	$-\dfrac{1}{k}\cos kx + c$
$\tan kx$	$\dfrac{1}{k}\ln\|\sec kx\|$+c
e^x	$\mathrm{e}^x + c$
e^{-x}	$-\mathrm{e}^{-x} + c$
e^{kx}	$\dfrac{1}{k}\mathrm{e}^{kx} + c$

When dealing with the trigonometric functions the variable x must always be measured in radians and not degrees. Note that the fourth entry in the table is valid for any value of n, positive, negative, or fractional, *except* $n = -1$. When $n = -1$ use the fifth entry in the table.

4. Computer Exercise or Activity

For this exercise it will be necessary for you to access the computer package DERIVE.
DERIVE can be used to obtain the indefinite integrals to most commonly occurring functions.

For example to find the indefinite integral of $\cos 3x$ you would key in Author:Expression $\cos(3x)$ followed by Calculus:Integrate. Then, in the Variable box choose x and in the Integral box choose Indefinite. On hitting the Simplify button DERIVE responds

$$\frac{\mathrm{SIN}(3 \cdot x)}{3}$$

Note that the constant of integration is usually omitted.

As a useful exercise use DERIVE to check the table of integrals on page 3. Note that the integral for x^n is presented as

$$\frac{x^{n+1}-1}{n+1}$$

which, up to a constant, is the correct expression.
Also note that DERIVE gives integrals involving the natural logarithm without using modulus signs: so that the indefinite integral of $\frac{1}{x}$ is presented as $\ln x$.

1. $x^2 - \mathrm{e}^x + c$ 2. $\frac{3}{2}\mathrm{e}^{2x} + c$ 3. $\frac{1}{6}x^2 + \frac{1}{6}\sin 2x + c$ 4. $-\frac{7}{x} + c$ 5. $\frac{1}{3}x^3 + 3x^2 + 9x + c$

Back to the theory