501

Algebra Questions

Introduction

This book is designed to provide you with review and practice for algebra success! It is not intended to teach common algebra topics. Instead, it provides 501 problems so you can flex your muscles and practice a variety of mathematical and algebraic skills. *501 Algebra Questions* is designed for many audiences. It's for anyone who has ever taken a course in algebra and needs to refresh and revive forgotten skills. It can be used to supplement current instruction in a math class. Or, it can be used by teachers and tutors who need to reinforce student skills. If, at some point, you feel you need further explanation about some of the algebra topics highlighted in this book, you can find them in the LearningExpress publication *Algebra Success in 20 Minutes a Day*.

How to Use This Book

First, look at the table of contents to see the types of algebra topics covered in this book. The book is organized into 20 chapters with a variety of arithmetic, algebra, and word problems. The structure follows a common sequence of concepts introduced in basic algebra courses. You may want to follow the sequence, as each succeeding chapter builds on skills taught in previous chapters. But if

P do operations inside *Parentheses*
E evaluate terms with *Exponents*
M D do *Multiplication* and *Division* in order from left to right
A S *Add* and *Subtract* terms in order from left to right

Tips for Working with Integers

<u>Addition</u>

Signed numbers the same? Find the SUM and use the same sign. Signed numbers different? Find the DIFFERENCE and use the sign of the larger number. (The larger number is the one whose value without a positive or negative sign is greatest.)

Addition is commutative. That is, you can add numbers in any order and the result is the same. As an example, $\mathbf{3 + 5 = 5 + 3}$, or $\mathbf{{}^{-}2 + {}^{-}1 = {}^{-}1 + {}^{-}2}$.

<u>Subtraction</u>

Change the operation sign to addition, change the sign of the number following the operation, then follow the rules for addition.

<u>Multiplication/Division</u>

Signs the same? Multiply or divide and give the result a **positive** sign. Signs different? Multiply or divide and give the result a **negative** sign.

Multiplication is commutative. You can multiply terms in any order and the result will be the same. For example: $\mathbf{(2 \cdot 5 \cdot 7) = (2 \cdot 7 \cdot 5) = (5 \cdot 2 \cdot 7) = (5 \cdot 7 \cdot 2)}$ and so on.

Evaluate the following expressions.

1. $27 + {}^{-}5$

2. ${}^{-}18 + {}^{-}20 - 16$

3. ${}^{-}15 - {}^{-}7$

4. $33 + {}^{-}16$

5. $8 + {}^{-}4 - 12$

6. $38 \div {}^{-}2 + 9$

7. ${}^{-}25 \cdot {}^{-}3 + 15 \cdot {}^{-}5$

8. ${}^{-}5 \cdot {}^{-}9 \cdot {}^{-}2$

9. $24 \cdot {}^{-}8 + 2$

10. $2 \cdot {}^{-}3 \cdot {}^{-}7$

11. ${}^{-}15 + 5 + {}^{-}11$

9. Group the terms being multiplied and evaluate. $(24 \cdot {}^-8) + 2$
Signs different? Multiply the terms and give the result a negative sign. $[24 \cdot {}^-8 = {}^-192]$
Substitute. $({}^-192) + 2$
Signs different? Subtract the value of the terms. $[192 - 2 = 190]$
Give the result the sign of the term with the larger value. $({}^-192) + 2 = {}^-190$
The simplified result of the numeric expression is as follows: $\underline{24 \cdot {}^-8 + 2 = {}^-190}$

10. Because all the operators are multiplication, you could group any two terms and the result would be the same. Let's group the last two terms. $2 \cdot ({}^-3 \cdot {}^-7)$
Signs the same? Multiply the terms and give the result a positive sign. $[({}^-3 \cdot {}^-7) = {}^+21]$
Substitute. $2 \cdot ({}^+21)$
Signs the same? Multiply the terms and give the result a positive sign. $2 \cdot ({}^+21) = {}^+42$
The simplified result of the numeric expression is as follows: $\underline{2 \cdot {}^-3 \cdot {}^-7 = {}^+42}$

11. Because all the operators are addition, you could group any two terms and the result would be the same. Or you could just work from left to right. $({}^-15 + 5) + {}^-11$
Signs different? Subtract the value of the numbers. $[15 - 5 = 10]$
Give the result the sign of the term with the larger value. $[({}^-15 + 5) = {}^-10]$
Substitute. $({}^-10) + {}^-11$
Signs the same? Add the value of the terms and give the result the same sign. $[10 + 11 = 21]$
$({}^-10) + {}^-11 = {}^-21$
The simplified result of the numeric expression is as follows: $\underline{{}^-15 + 5 + {}^-11 = {}^-21}$

12. First evaluate the expressions within the parentheses. $[49 \div 7 = 7]$
Signs different? Divide and give the result a negative sign. $[48 \div {}^-4 = {}^-12]$
Substitute into the original expression. $(7) - ({}^-12)$
Change the subtraction sign to addition by changing the sign of the number that follows it. $7 + {}^+12$
Signs the same? Add the value of the terms and give the result the same sign. $7 + {}^+12 = {}^+19$
The simplified result of the numeric expression is as follows: $\underline{(49 \div 7) - (48 \div {}^-4) = {}^+19}$

15. First evaluate the expressions within the parentheses. $[(^{-}18 \div 2)]$

Signs different? Divide the value of the terms and give the result a negative sign. $[18 \div 2 = 9]$

$[(^{-}18 \div 2 = {}^{-}9)]$

Signs different? Multiply the term values and give the result a negative sign. $(6 \cdot {}^{-}3)$

$[6 \cdot 3 = 18]$

$(6 \cdot {}^{-}3) = {}^{-}18$

Substitute the values into the original expression. $(^{-}9) - (^{-}18)$

Change subtraction to addition and change the sign of the term that follows. $(^{-}9) + (^{+}18)$

Signs different? Subtract the value of the numbers and give the result the sign of the higher value number. $[18 - 9 = 9]$

$(^{-}9) + (^{+}18) = {}^{+}9$

The simplified result of the numeric expression is as follows: $\underline{(^{-}18 \div 2) - (6 \cdot {}^{-}3) = {}^{+}9}$

16. Evaluate the expressions within the parentheses. $(64 \div {}^{-}16)$

Signs different? Divide and give the result a negative sign. $[64 \div 16 = 4]$

$(64 \div {}^{-}16 = {}^{-}4)$

Substitute the value into the original expression. $23 + (^{-}4)$

Signs different? Subtract the value of the numbers and give the result the sign of the higher value number. $[23 - 4 = 19]$

$23 + (^{-}4) = {}^{+}19$

The simplified result of the numeric expression is as follows: $\underline{23 + (64 \div {}^{-}16) = {}^{+}19}$

17. The order of operations tells us to evaluate the terms with exponents first. $[2^3 = 2 \cdot 2 \cdot 2 = 8]$

$[(^{-}4)^2 = (^{-}4) \cdot (^{-}4)]$

Signs the same? Multiply the terms and give the result a positive sign. $[4 \cdot 4 = 16]$

$[(^{-}4)^2 = {}^{+}16]$

Substitute the values of terms with exponents into the original expression. $2^3 - (^{-}4)^2 = (8) - (^{+}16)$

Change subtraction to addition and change the sign of the term that follows. $8 + {}^{-}16$

Signs different? Subtract the value of the numbers and give the result the sign of the higher value number. $[16 - 8 = 8]$

$8 + {}^{-}16 = {}^{-}8$

49. Substitute the values for the variables into the expression.	$(^-8)^2 - 4(3)^2(\frac{1}{2})$
Evaluate the first term.	$[(^-8)^2 = {^-8} \cdot {^-8}]$
Signs the same? Multiply and give the result a positive sign.	$[^-8 \cdot {^-8} = 64]$
Evaluate the second term.	$[4(3)^2(\frac{1}{2}) = 4 \cdot 3 \cdot 3 \cdot \frac{1}{2}]$
Multiply from left to right.	$[4 \cdot 3 = 12]$
	$[12 \cdot 3 = 36]$
	$[36 \cdot \frac{1}{2} = 18]$
Substitute the results into the numerical expression.	$(64) - (18)$
Yes, you can just subtract.	$64 - 18 = 46$
The simplified value of the expression is as follows:	$\underline{z^2 - 4a^2y = 46}$
50. Substitute the values for the variables into the expression.	$3(6)^2(^-5)(5(3) - 3(^-5))$
PEMDAS: Evaluate the expression in the parentheses first.	$[(5(3) - 3(^-5)) = 5 \cdot 3 - 3 \cdot {^-5}]$
	$[5 \cdot 3 - 3 \cdot {^-5} = 15 - {^-15}]$
Change subtraction to addition and the sign of the term that follows.	$[15 + {^+15} = 30]$
Substitute the result into the numerical expression.	$3(6)^2(^-5)(30)$
PEMDAS: Evaluate terms with exponents next.	$[(6)^2 = 6 \cdot 6 = 36]$
Substitute the results into the numerical expression.	$3(36)(^-5)(30)$
Multiply from left to right.	$[3(36) = 108]$
Signs different? Multiply the values and give a negative sign.	$[(108) \cdot (^-5) = {^-540}]$
	$[(^-540) \cdot (30) = {^-16{,}200}]$
	$3(6)^2(^-5)(5(3) - 3(^-5)) = {^-16{,}200}$
The simplified value of the expression is as follows:	$\underline{3x^2b(5a - 3b) = {^-16{,}200}}$

Associative Property of Addition

$(q + r) + s = q + (r + s)$

This equation reminds us that when you are performing a series of additions of terms, you can associate any term with any other and the result will be the same.

Associative Property of Multiplication

$(d \cdot e) \cdot f = d \cdot (e \cdot f)$

This equation reminds us that you can multiply three or more terms in any order without changing the value of the result.

Identity Property of Addition

$n + 0 = n$

Identity Property of Multiplication

$n \cdot 1 = n$

Term Equivalents

$x = 1 \cdot x$

For purposes of combining like terms, a variable by itself is understood to mean one of that term.

$n = {}^{+}n$

A term without a sign in front of it is considered to be positive.

$a + {}^{-}b = a - {}^{+}b = a - b$

Adding a negative term is the same as subtracting a positive term. Look at the expressions on either side of the equal signs. Which one looks simpler? Of course, it's the last, $a - b$. Clarity is valued in mathematics. Writing expressions as simply as possible is always appreciated.

While it may not seem relevant yet, as you go through the practice exercises, you will see how each of these properties will come into play as we simplify algebraic expressions by combining like terms.

57. Use the distributive property of multiplication on the first term. $[3(2a + 3b) = 3 \cdot 2a + 3 \cdot 3b]$
$[6a + 9b]$
Use the distributive property of multiplication on the second term. $[7(a - b) = 7 \cdot a - 7 \cdot b]$
$[7a - 7b]$
Substitute the results into the expression. $(6a + 9b) + (7a - 7b)$
$6a + 9b + 7a - 7b$
Change subtraction to addition and change the sign of the term that follows. $6a + 9b + 7a + ({}^{-}7b)$
Use the commutative property for addition to put like terms together. $6a + 7a + 9b + ({}^{-}7b)$
Use the associative property for addition. $(6a + 7a) + (9b + {}^{-}7b)$
Add like terms. $[6a + 7a = 13a]$
Signs different? Subtract the value of the terms. $[9b + {}^{-}7b = 2b]$
Substitute the result into the expression. $(13a) + (2b)$
The simplified algebraic expression is: $\underline{13a + 2b}$

58. Use the distributive property of multiplication on the first term. $[11(4m + 5) = 11 \cdot 4m + 11 \cdot 5]$
$[44m + 55]$
Use the distributive property of multiplication on the second term. $[3({}^{-}3m + 8) = 3 \cdot {}^{-}3m + 3 \cdot 8]$
$[{}^{-}9m + 24]$
Substitute the results into the expression. $(44m + 55) + ({}^{-}9m + 24)$
$44m + 55 + {}^{-}9m + 24$
Use the commutative property for addition to put like terms together. $44m + {}^{-}9m + 55 + 24$
Use the associative property for addition. $(44m + {}^{-}9m) + (55 + 24)$
Add like terms. $[44m + {}^{-}9m = 35m]$
$[55 + 24 = 79]$
Substitute the result into the expression. $(35m) + (79)$
The simplified algebraic expression is: $\underline{35m + 79}$

59. Use the distributive property of multiplication on the second term. $[5(n - 8) = 5 \cdot n - 5 \cdot 8]$
$[5n - 40]$
Substitute the result into the expression. $64 + (5n - 40) + 12n - 24$
Parentheses are no longer needed. $64 + 5n - 40 + 12n - 24$
Change subtraction to addition and change the sign of the term that follows. $64 + 5n + {}^{-}40 + 12n + {}^{-}24$

71.

Change subtraction to addition and the sign of the terms that follow.	$5(3x + {}^-y) + x(5 + 2y) + {}^-4(3 + x)$
Use the distributive property of multiplication on the first term.	$[5(3x + {}^-y) = 5 \cdot 3x + 5 \cdot {}^-y]$
Use the rules for multiplying signed terms.	$[5 \cdot 3x + 5 \cdot {}^-y = 15x + {}^-5y]$
Use the distributive property of multiplication on the second term.	$[x(5 + 2y) = x \cdot 5 + x \cdot 2y]$
Use the rules for multiplying signed terms.	$[x \cdot 5 + x \cdot 2y = 5x + 2xy]$
Use the distributive property of multiplication on the third term.	$[{}^-4(3 + x) = {}^-4 \cdot 3 + {}^-4 \cdot x]$
Use the rules for multiplying signed terms.	$[{}^-4 \cdot 3 + {}^-4 \cdot x = {}^-12 + {}^-4x]$
Substitute the results into the original expression.	$(15x + {}^-5y) + (5x + 2xy) + ({}^-12 + {}^-4x)$
Remove the parentheses.	$15x + {}^-5y + 5x + 2xy + {}^-12 + {}^-4x$
Use the commutative property of addition to move like terms together. Use the associative property for addition.	$(15x + 5x + {}^-4x) + {}^-5y + 2xy + {}^-12$
Combine like terms using addition rules for signed numbers.	$(16x) + {}^-5y + 2xy + {}^-12$
Adding a negative term is the same as subtracting a positive term.	$(16x) - ({}^+5y) + 2xy - ({}^+12)$
Remove the parentheses.	$\underline{16x - 5y + 2xy - 12}$

4

Solving Basic Equations

Solving equations is not very different from working with numerical or algebraic expressions. An equation is a mathematical statement where two expressions are set equal to each other. Using logic and mathematical operations, you can manipulate the equation to find a solution. Simply put, that is what you have done when you have the variable on one side of the equal sign and a number on the other. The answer explanations will show and identify all the steps you will need to solve basic equations. There will be different solutions to similar problems to show a variety of methods for solving equations. But they all rely on the same rules. Look over the **Tips for Solving Basic Equations** before you begin this chapter's questions.

Tips for Solving Basic Equations

- If a number is being added to or subtracted from a term on one side of an equation, you can eliminate that number by performing the inverse operation.
- The inverse of addition is subtraction. The inverse of subtraction is addition. If you add or subtract an amount from one side of the equation, you must do the same to the other side to maintain the equality.

105. Subtract 4 from both sides of the equation.	$\frac{x}{3} + 4 - 4 = 10 - 4$
Associate like terms.	$\frac{x}{3} + (4 - 4) = 10 - 4$
Perform numerical operations.	$\frac{x}{3} + (0) = 6$
Zero is the identity element for addition.	$\frac{x}{3} = 6$
Multiply both sides of the equation by 3.	$3(\frac{x}{3}) = 3(6)$
	$\underline{x = 18}$

106. Add 5 to each side of the equation.	$\frac{x}{7} - 5 + 5 = 1 + 5$
Change subtraction to addition and change the sign of the term that follows.	$\frac{x}{7} + {}^{-}5 + 5 = 1 + 5$
Associate like terms.	$\frac{x}{7} + ({}^{-}5 + 5) = 1 + 5$
Perform numerical operations.	$\frac{x}{7} + (0) = 6$
Zero is the identity element for addition.	$\frac{x}{7} = 6$
Multiply both sides of the equation by 7.	$7(\frac{x}{7}) = 7(6)$
	$\underline{x = 42}$

107. Add 9 to each side of the equation.	$39 + 9 = 3a - 9 + 9$
Change subtraction to addition and change the sign of the term that follows.	$39 + 9 = 3a + ({}^{-}9 + 9)$
Associate like terms.	$39 + 9 = 3a + ({}^{-}9 + 9)$
Perform numerical operations.	$48 = 3a + (0)$
Zero is the identity element for addition.	$48 = 3a$
Divide both sides of the equation by 3.	$48 \div 3 = 3a \div 3$
	$\underline{16 = a}$

108. Subtract 20 from both sides of the equation.	$4 - 20 = 4a + 20 - 20$
Associate like terms.	$4 - 20 = 4a + (20 - 20)$
Perform numerical operations.	${}^{-}16 = 4a + (0)$
Zero is the identity element for addition.	${}^{-}16 = 4a$
Divide both sides of the equation by 4.	$\frac{{}^{-}16}{4} = \frac{4a}{4}$
	$\underline{{}^{-}4 = a}$

109. Subtract 5 from both sides of the equation.	$10a + 5 - 5 = 7 - 5$
Associate like terms.	$10a + (5 - 5) = 7 - 5$
Perform numerical operations.	$10a + (0) = 2$
Zero is the identity element for addition.	$10a = 2$
Divide both sides of the equation by 10.	$\frac{10a}{10} = \frac{2}{10}$
	$\underline{a = 0.2 \text{ or } \frac{1}{5}}$

6

Solving Equations with Variables on Both Sides of an Equation

If you have been solving the problems in this book with some success, you will move easily into this chapter. Work through the questions carefully, and refer to the answer explanations as you try and solve the equations by yourself. Then check your answers with the solutions provided. If your sequence of steps is not identical to the solution shown, but you are getting the correct answers, that's all right. There is often more than one way to find a solution. And it demonstrates your mastery of the processes involved in doing algebra.

Tips for Solving Equations with Variables on Both Sides of the Equation

Use the distributive property of multiplication to expand and separate terms. Notice that what follows are variations on the basic distributive property.

Combine like terms on each side of the equation.	$21x = {}^{-}21$
Divide both sides of the equation by 21.	$\frac{21x}{21} = \frac{^{-}21}{21}$
Simplify the expression.	$\underline{x = {}^{-}1}$

140.

Use the distributive property of multiplication.	$2(2x) + 2(19) - 9x = 9(13) - 9(x) + 21$
Simplify the expression.	$4x + 38 - 9x = 117 - 9x + 21$
Use the commutative property with like terms.	$38 + 4x - 9x = 117 + 21 - 9x$
Combine like terms on each side of the equation.	$38 - 5x = 138 - 9x$
Subtract 38 from both sides of the equation.	$38 - 38 - 5x = 138 - 38 - 9x$
Combine like terms on each side of the equation.	${}^{-}5x = 100 - 9x$
Add $9x$ to both sides of the equation.	${}^{-}5x + 9x = 100 - 9x + 9x$
Combine like terms on each side of the equation.	$4x = 100$
Divide both sides of the equation by 4.	$\frac{4x}{4} = \frac{100}{4}$
Simplify the expression.	$\underline{x = 25}$
Now let's check the answer by substituting the solution into the original equation.	$2(2(25) + 19) - 9(25) = 9(13 - (25)) + 21$
Simplify the expression. Use order of operations.	$2(50 + 19) - 225 = 9({}^{-}12) + 21$
	$2(69) - 225 = {}^{-}108 + 21$
	$138 - 225 = {}^{-}87$
	${}^{-}87 = {}^{-}87$
The solution is correct.	

7

Using Formulas to Solve Equations

Chances are you have been asked to use formulas to solve problems in math, science, social studies, or technology. Algebra is a useful skill to know when faced with problems in these areas. In this chapter, you will have the chance to solve word problems that could pop up in any one of these subject areas. These word problems require you to find an unknown value in a formula. You will be using your algebra problem-solving skills in every problem.

Tips for Using Formulas to Solve Equations

Given a formula with several variables, you will generally be given values for all but one. Then you will be asked to solve the equation for the missing variable. It can be helpful to list each variable with its given value. Put a question mark next to the equal sign in place of the value for the unknown variable.

Keep in mind the rules for order of operations.

Select from these formulas the appropriate one to solve the following word problems:

Volume of a rectangular solid: $V = lwh$ where l = length, w = width, h = height

8

Graphing Linear Equations

This chapter asks you to find solutions to linear equations by graphing. The solution of a linear equation is the set of ordered pairs that form a line on a coordinate graph. Every point on the line is a solution for the equation. One method for graphing the solution is to use a table with x and y values that are solutions for the particular equation. You select a value for x and solve for the y value. But in this chapter, we will focus on the slope and y-intercept method.

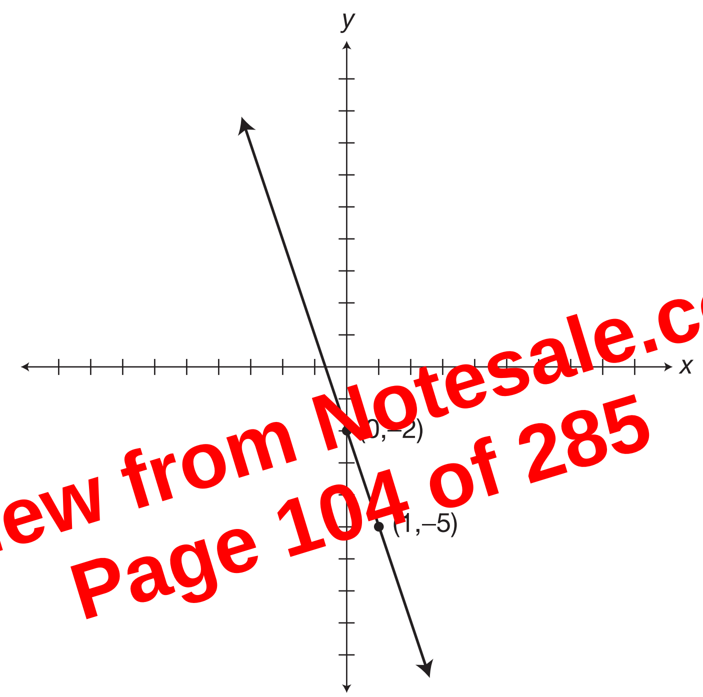
The slope and y-intercept method may require you to change an equation into the slope-intercept form. That is, the equation with two variables must be written in the form $y = \boldsymbol{m}x + \boldsymbol{b}$. Written in this form, the $\boldsymbol{m}$ value is a number that represents the slope of the solution graph and the $\boldsymbol{b}$ is a number that represents the y-intercept. The slope of a line is the ratio of the change in the y value over the change in the x value from one point on the solution graph to another. From one point to another, the slope is the rise over the run. The y-intercept is the point where the solution graph (line) crosses the y-axis. Another way of saying that is: The y-intercept is the place where the value of x is 0.

Tips for Graphing Linear Equations

- Rewrite the given equation in the form $y = mx + b$.
- Use the $\boldsymbol{b}$ value to determine where the line crosses the y-axis. That is the point $(0,b)$.

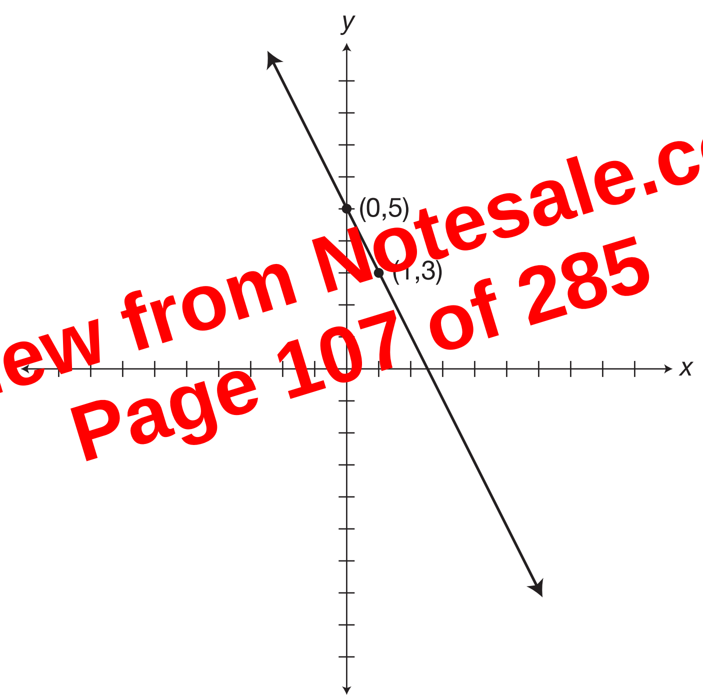
177. Put the equation in the proper form.

Subtract $3x$ from both sides of the equation.	$y + 3x - 3x = {}^{-}3x - 2$
Simplify the equation.	$y = {}^{-}3x - 2$
The equation is in the proper slope/y-intercept form.	$m = {}^{-}3 = \frac{-3}{1} = \frac{\text{change in } y}{\text{change in } x}$
$b = {}^{-}2$.	The y-intercept is at the point $(0,{}^{-}2)$.
A change in y of ${}^{-}3$ and in x of 1 gives the point	$(0 + 1,{}^{-}2 - 3)$ or $(1,{}^{-}5)$.



180. Put the equation in the proper form.

Subtract $4x$ from both sides of the equation.	$2y + 4x - 4x = {}^{-}4x + 10$
Simplify the equation.	$2y = {}^{-}4x + 10$
Divide both sides of the equation by 2.	$\frac{2y}{2} = \frac{({}^{-}4x + 10)}{2}$
Simplify the equation.	$y = \frac{{}^{-}4x}{2} + \frac{10}{2}$
	$y = {}^{-}2x + 5$
The equation is in the proper slope/y-intercept form.	$m = {}^{-}2 = -\frac{2}{1} = \frac{\text{change in } y}{\text{change in } x}$
$b = 5$.	The y-intercept is at the point (0,5).
A change in y of ${}^{-}2$ and in x of 1 gives the point	$(0 + 1, 5 - 2)$ or (1,3).



181. Put the equation in the proper form.

Add $3y$ to both sides of the equation. $x - 3y + 3y = 12 + 3y$

Simplify the equation. $x = 12 + 3y$

Subtract 12 from both sides of the equation. $x - 12 = 12 - 12 + 3y$

Simplify the equation. $x - 12 = 3y$

Divide both sides of the equation by 3. $\frac{(x - 12)}{3} = y$

Simplify the equation. $\frac{x}{3} - \frac{12}{3} = y$

$\frac{x}{3} - 4 = y$

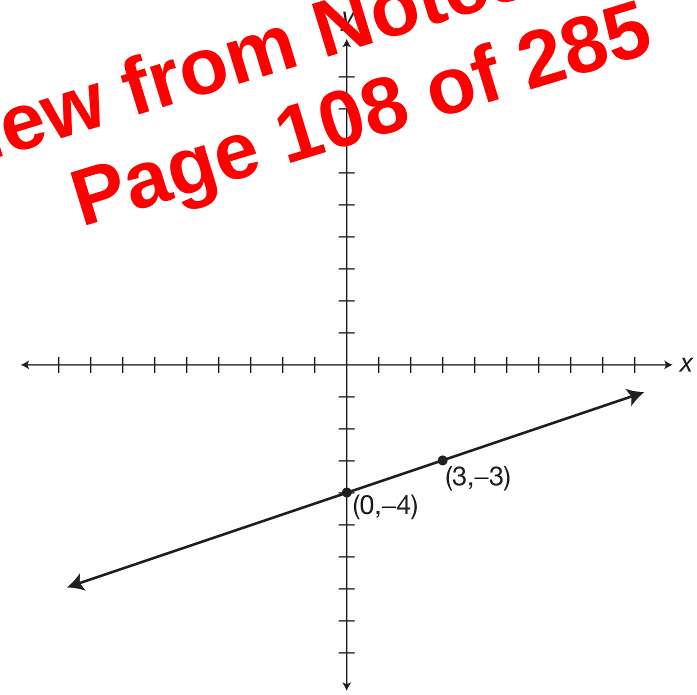
$\frac{x}{3} = \frac{(1)(x)}{(3)(1)} = \frac{1}{3} \cdot \frac{x}{1} = \frac{1}{3}x$ $\quad \frac{1}{3}x - 4 = y$

The equation is equivalent to the proper form. $y = \frac{1}{3}x - 4$

The equation is in the proper slope/y-intercept form. $m = \frac{1}{3} = \frac{\text{change in } y}{\text{change in } x}$

$b = {}^{-}4$. The y-intercept is at the point $(0,{}^{-}4)$.

A change in y of 1 and in x of 3 gives the point $(0 + 3,{}^{-}4 + 1)$ or $(3,{}^{-}3)$.



182. Put the equation in the proper form.

Divide both sides of the equation by 3. $\frac{(3x + 9y)}{3} = \frac{^-27}{3}$

Simplify the equation. $\frac{3x}{3} + \frac{9y}{3} = {^-9}$

$x + 3y = {^-9}$

Subtract x from both sides of the equation. $x - x + 3y = {^-x} - 9$

Simplify the equation. $3y = {^-x} - 9$

Divide both sides of the equation by 3. $\frac{3y}{3} = \frac{(^-x - 9)}{3}$

Simplify the equation. $y = \frac{^-x}{3} - \frac{9}{3}$

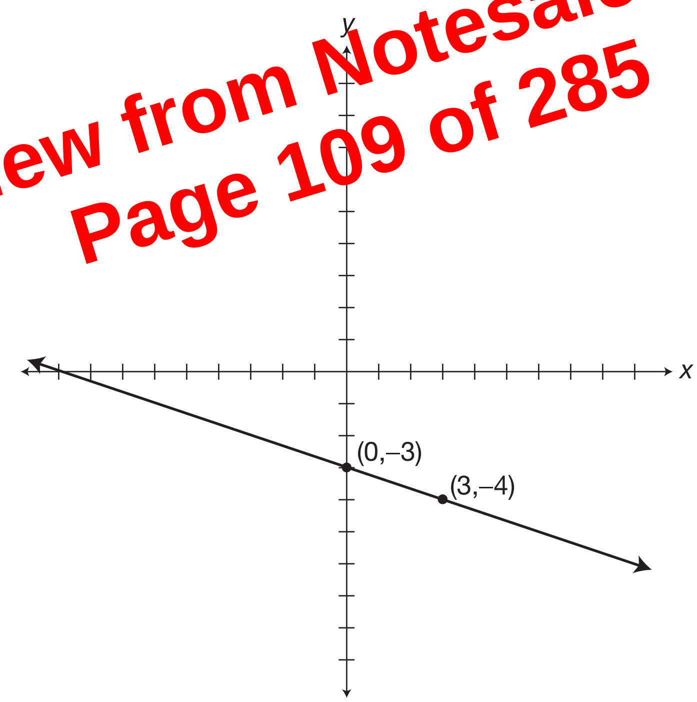
$y = \frac{^-x}{3} - \frac{^-9}{3}$

$y = \frac{^-1}{3x} - 3$

The equation is in the proper slope/y-intercept form. $m = -\frac{1}{3} = \frac{\text{change in } y}{\text{change in } x}$

$b = {^-3}$. The y-intercept is at the point $(0,{^-3})$.

A change in y of $^-1$ and in x of 3 gives the point $(0 + 3,{^-3} - 1)$ or $(3,{^-4})$.



183. Put the equation in the proper form.

Add $5x$ to both sides of the equation. $5x - 5x - y = 5x - \frac{7}{2}$

Simplify the equation. ${}^{-}y = 5x - \frac{7}{2}$

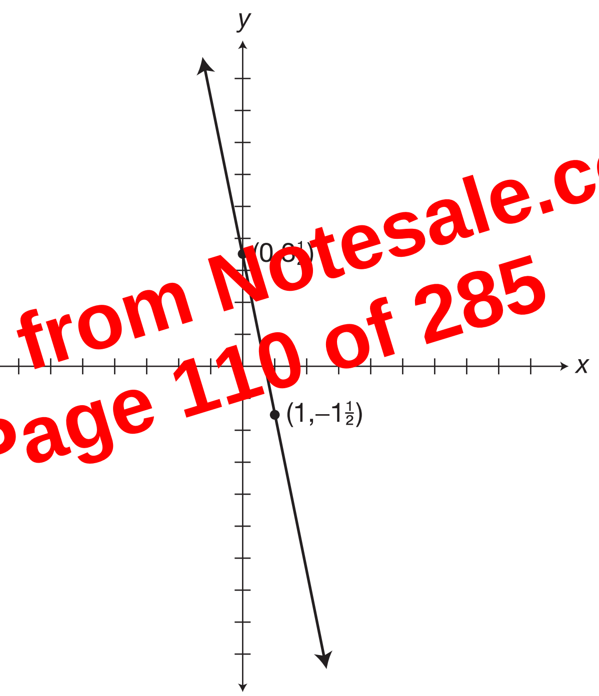
Multiply both sides of the equation by ${}^{-}1$. ${}^{-}1({}^{-}y) = {}^{-}1(5x - \frac{7}{2})$

Simplify the equation. $y = {}^{-}5x + \frac{7}{2}$

The equation is in the proper slope/y-intercept form. $m = {}^{-}5 = -\frac{5}{1} = \frac{\text{change in } y}{\text{change in } x}$

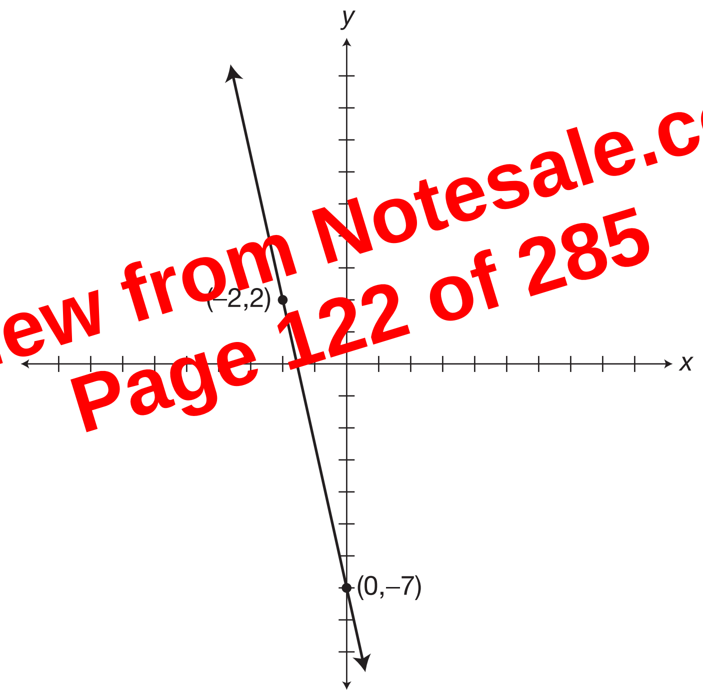
$b = \frac{7}{2} = 3\frac{1}{2}$. The y-intercept is at the point $(0,3\frac{1}{2})$.

A change in y of ${}^{-}5$ and in x of 1 gives the point of $(0 + 1,3\frac{1}{2} - 5)$ or $(1,{}^{-}1\frac{1}{2})$.



195. Put the equation in the proper form.

Add ^-27x to both sides of the equation.	$6y + 27x - 27x = {}^-27x - 42$
Simplify the equation.	$6y = {}^-27x - 42$
Divide both sides of the equation by 6.	$\frac{6y}{6} = \frac{^-27x}{6} - \frac{42}{6}$
Simplify the equation.	$y = \frac{^-27}{6}x - 7$
Simplify the coefficient of x by a common factor of 3.	$y = \frac{^-9}{2}x - 7$
The equation is in the proper slope/y-intercept form.	$m = \frac{^-9}{2} = \frac{9}{^-2} = \frac{\text{change in } y}{\text{change in } x}$
$b = -7$.	The y-intercept is at the point $(0,{}^-7)$.
A change in y of 9 and in x of $^-2$ gives the point	$(0 - 2,{}^-7 + 9)$ or $(^-2,2)$.



196. Let x = horizontal movement. Forward is in the positive direction.
Let y = vertical movement. Ascending is in the positive direction. Descending is in the negative.
The change in position of the glider is described by the slope.

The change in y is $^-1$ for every change in x of $^+25$.	Slope $= \frac{\text{change in } y}{\text{change in } x} = \frac{^-1}{25} = m$

The starting position for the purposes of this graphic solution is at an altitude of 250 ft or $^{+}250$. So: $b = 250$

Using the standard form $y = mx + b$, you substitute the given values into the formula. $\underline{y = \frac{^{-}1}{25}x + 250}$

A graph of this equation would have a slope of $^{-}\frac{1}{25}$ and the y-intercept would be at $(0,250)$

197. Let y = the amount of a monthly bill.
Let x = the hours of Internet use for the month.
The costs for the month will equal \$15 plus the \$.25 times the number of hours of use.
Written as an equation, this information would be as follows: $\underline{y = 0.25x + 15}$
A graph of this equation would have a slope of 0.25 or $\frac{25}{100} = \frac{1}{4}$
The y-intercept would be at $(0,15)$

198. Let y = the cost of a scooter rental for one day.
Let x = the number of miles driven in one day.
The problem tells us that the cost would be equal to the daily charges plus the 0.05 times the number of miles driven.
Written as an equation, this would be: $\underline{y = 0.05x + 20}$
The graph would have a y-intercept at (0,20) and the slope would be $\frac{5}{100} = \frac{1}{20}$

199. Let y = the total cost for equipment.
Let x = the number of tanks used during the week.
The problem tells us that the cost would be equal to the weekly charge for gear rental plus 8 times the number of tanks used.
A formula that would represent this information would be: $\underline{y = 8x + 150}$
The y-intercept would be at (0,150) and the slope $= 8 = \frac{8}{1}$.

200. Let y = the number of birds that visited a backyard feeder.
Let x = the number of chickadees that visited the feeder.
An equation that represents the statement would be: $\underline{y = 7x}$
The y-intercept is (0,0) and the slope $= 7 = \frac{7}{1}$.

217.	You can simplify equations (and inequalities) with fractions by multiplying them by a common multiple of the denominators.	
	Multiply both sides of the inequality by 4.	$4(x - \frac{3}{4}) < 4(\frac{^-3}{4}(x + 2))$
	Use the distributive property of multiplication.	$4(x) - 4(\frac{3}{4}) < 4(\frac{^-3}{4})(x + 2)$
	Simplify the expressions.	$4x - 3 < {}^-3(x + 2)$
	Use the distributive property of multiplication.	$4x - 3 < {}^-3(x) - 3(2)$
	Simplify.	$4x - 3 < {}^-3x - 6$
	Add 3 to both sides of the inequality.	$4x - 3 + 3 < {}^-3x - 6 + 3$
	Combine like terms.	$4x < -\ 3x - 3$
	Add 3x to both sides of the equation.	$3x + 4x < 3x - 3x - 3$
	Combine like terms.	$7x < {}^-3$
	Divide both sides of the inequality by 7.	$\frac{7x}{7} < \frac{^-3}{7}$
	Sure, you have a fraction for an answer, but it can be easier to operate with whole numbers until the last step.	
	Simplify the expressions.	$x < \frac{^-3}{7}$
218.	Subtract 0.1 from both sides of the inequality.	$\frac{3}{2}x + 0.1 - 0.1 \geq 0.9 - 0.1 + x$
	Combine like terms on each side of the inequality.	$\frac{3}{2}x \geq 0.8 + x$
	Subtract x from both sides of the inequality.	$\frac{3}{2}x - x \geq 0.8 + x - x$
	Simplify.	$\frac{1}{2}x \geq 0.8$
	Multiply both sides of the inequality by 2.	$2(\frac{1}{2}x) \geq 2(0.8)$
	Simplify the expressions.	$x \geq 1.6$
219.	Change the term to an improper fraction.	$x - \frac{13}{3} < 9 + \frac{2}{3}x$
	Multiply both sides of the inequality by 3.	$3(x - \frac{13}{3}) < 3(9 + \frac{2}{3}x)$
	Use the distributive property of multiplication.	$3(x) - 3(\frac{13}{3}) < 3(9) + 3(\frac{2}{3}x)$
	Simplify the terms.	$3x - 13 < 27 + 2x$
	Add 13 to both sides of the inequality.	$3x - 13 + 13 < 13 + 27 + 2x$
	Combine like terms and simplify.	$3x < 40 + 2x$
	Subtract 2x from both sides of the inequality.	$3x - 2x < 40 + 2x - 2x$
	Simplify.	$x < 40$

10

Graphing Inequalities

In this chapter, you will practice graphing inequalities that have one or two variables. When there is only one variable, you use a number line. When there are two variables, you use a coordinate plane.

Tips for Graphing Inequalities

When using a number line to show the solution graph for an inequality, use a solid circle on the number line as the endpoint when the inequality symbol is ≤ or ≥. When the inequality symbol is < or >, use an open circle to show the endpoint. A solid circle shows that the solution graph includes the endpoint; an open circle shows that the solution graph does not include the endpoint.

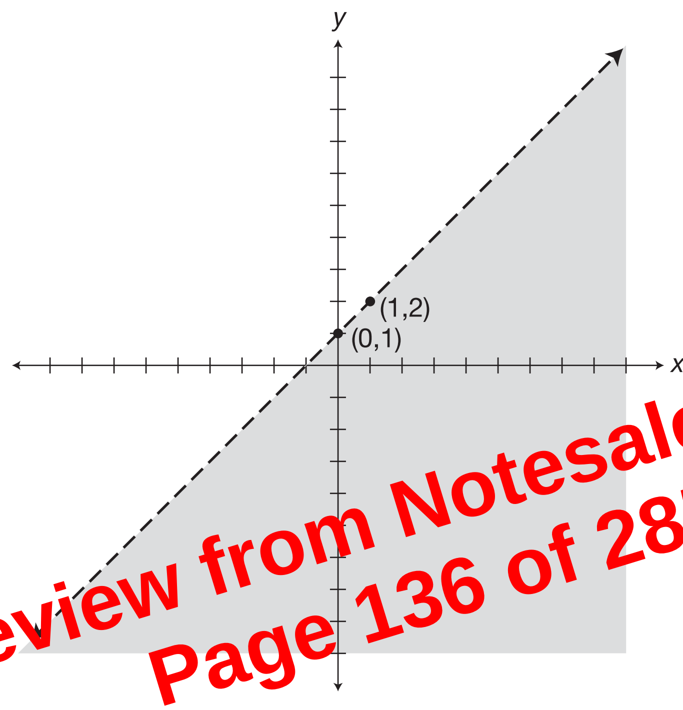
When there are two variables, use a coordinate plane to graph the solution. Use the skills you have been practicing in the previous chapters to transform the inequality into the slope/*y*-intercept form you used to graph equalities with two variables. Use the slope and the *y*-intercept of the transformed inequality to show the boundary line for your solution graph. Draw a solid line when the inequality symbol is ≤ or ≥. Draw a dotted line when the inequality symbol is < or >. To complete the graph, shade the region above the boundary line when the inequality symbol is > or ≥. Shade the region below the boundary line when the inequality symbol is < or ≤.

231. The inequality is in the proper slope/y-intercept form. $m = 1 = \frac{1}{1} = \frac{\text{change in } y}{\text{change in } x}$

$b = 1$. The y-intercept is at the point (0,1).

A change in y of 1 and in x of 1 gives the point (0 + 1,1 + 1) or (1,2).

Draw a dotted boundary line and shade below it.



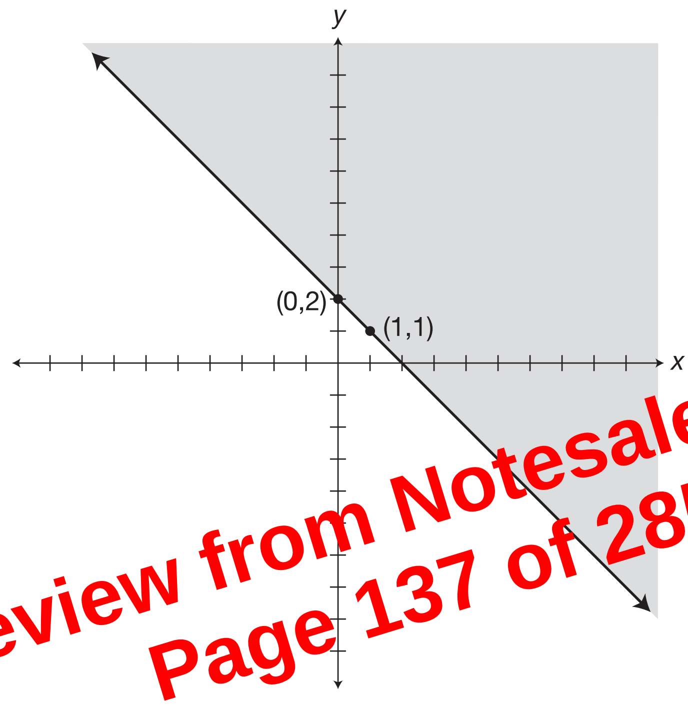
232. The inequality is in the proper slope/y-intercept form.

$m = {}^{-}1 = \frac{{}^{-}1}{1} = \frac{\text{change in } y}{\text{change in } x}$

$b = 2$. The y-intercept is at the point (0,2).

A change in y of ${}^{-}1$ and in x of 1 gives the point (0 + 1,2 – 1) or (1,1).

Draw a solid boundary line and shade above it.



Answers

Numerical expressions in parentheses like this [ ] are operations performed on only part of the original expression. The operations performed within these symbols are intended to show how to evaluate the various terms that make up the entire expression.

Expressions with parentheses that look like this ( ) contain either numerical substitutions or expressions that are part of a numerical expression. Once a single number appears within these parentheses, the parentheses are no longer needed and need not be used the next time the entire expression is written.

When two pair of parentheses appear side by side like this ( )( ), it means that the expressions within are to be multiplied.

Sometimes parentheses appear within other parentheses in numerical or algebraic expressions. Regardless of what symbol is used, ( ), { }, or [ ], perform operations in the innermost parentheses first and work outward.

The underlined ordered pair is the solution. The graph is shown.

251. Transform equations into slope/y-intercept form.
The equation is in the proper slope/y-intercept form.

$y = x + 4$

$b = 4$.

$m = 1 = \frac{1}{1} = \frac{\text{change in } y}{\text{change in } x}$

The y-intercept is at the point (0,4).

The slope tells you to go up 1 space and right 1 for (1,5).

* * * * * * * * * * * *

$y = {}^{-}x + 2$

The equation is in the proper slope/y-intercept form.

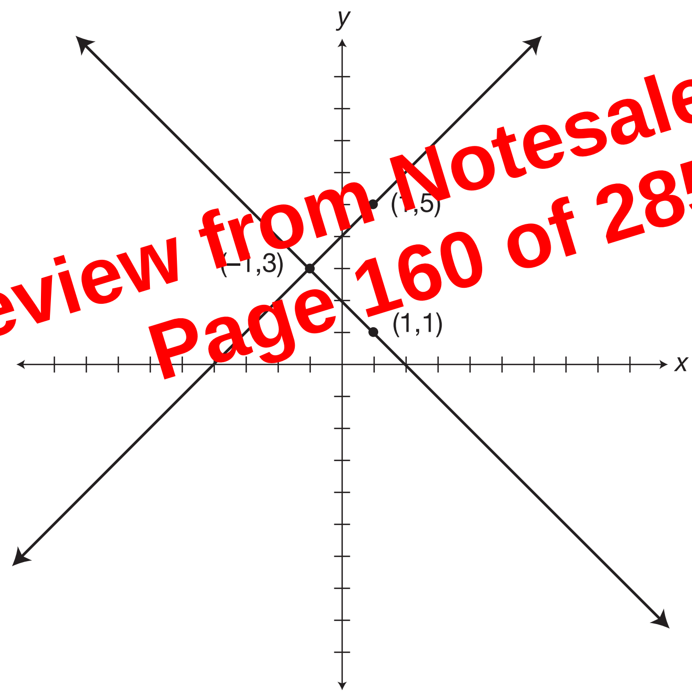
$m = {}^{-}1 = \frac{-1}{1} = \frac{\text{change in } y}{\text{change in } x}$

$b = 2$.

The y-intercept is at the point (0,2).

The slope tells you to go down 1 space and right 1 for (1,1).

The solution is ($^{-}$1,3).



255. Transform equations into slope/y-intercept form. $2y = 6x + 14$

Divide both sides by 2. $y = \frac{6}{2}x + 7$

The equation is in the proper slope/y-intercept form.

Use the negatives to keep the coordinates near the origin. $m = \frac{6}{2} = \frac{^-6}{^-2} = \frac{\text{change in } y}{\text{change in } x}$

$b = 7$. The y-intercept is at the point (0,7).

The slope tells you to go down 6 spaces and left 2 for ($^-$2,1).

* * * * * * * * * * * *

$4y = x - 16$

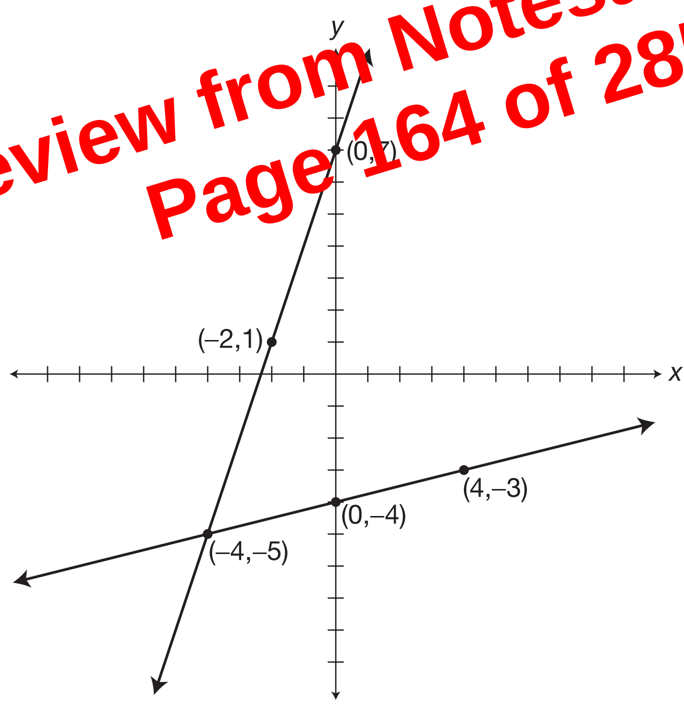
Divide both sides by 4. $y = \frac{1}{4}x - 4$

The equation is in the proper slope/y-intercept form. $m = \frac{1}{4} = \frac{\text{change in } y}{\text{change in } x}$

$b = {}^-4$. The y-intercept is at the point (0,$^-$4).

The slope tells you to go up 1 space and right 4 for (4,$^-$3).

The solution for the system of equations is ($^-$4,$^-$5).



261. Transform equations into slope/y-intercept form. $3y = 6x + 6$

Divide both sides by 3. $y = 2x + 2$

The equation is in the proper slope/y-intercept form. $m = 2 = \frac{2}{1} = \frac{\text{change in } y}{\text{change in } x}$

$b = 2$. The y-intercept is at the point (0,2).

The slope tells you to go up 2 spaces and right 1 for (1,4).

* * * * * * * * * * * *

$5y = 10(x - 5)$

Use the distributive property. $5y = 10x - 50$

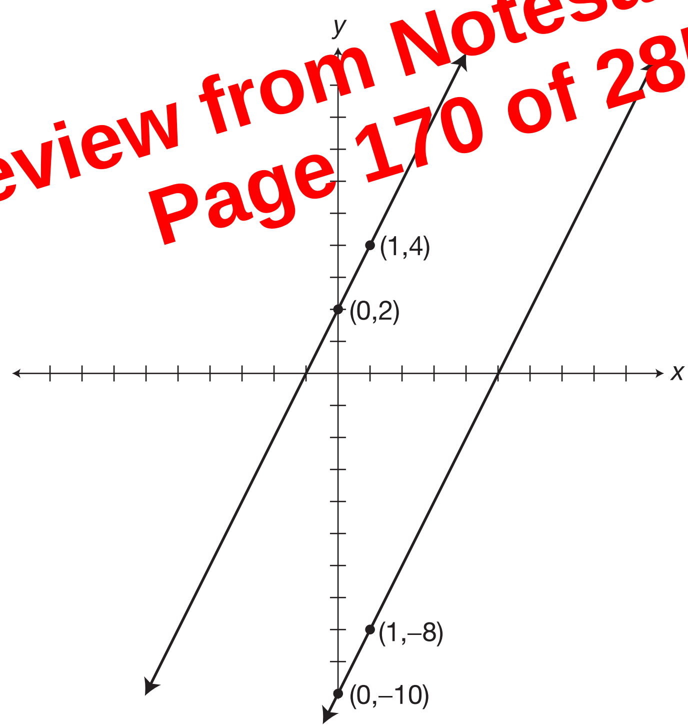
Divide both sides by 5. $y = 2x - 10$

The equation is in the proper slope/y-intercept form. $m = 2 = \frac{2}{1} = \frac{\text{change in } y}{\text{change in } x}$

$b = {}^{-}10$. The y-intercept is at the point $(0,{}^{-}10)$.

The slope tells you to go up 2 spaces and right 1 for $(1,{}^{-}8)$.

The slopes are the same, so the line graphs are parallel and do not intersect.



266. Transform the inequalities into slope/y-intercept form. $2y - 3x \geq {}^{-}6$

Add $3x$ to both sides. $2y - 3x + 3x \geq {}^{+}3x - 6$

Simplify. $2y \geq 3x - 6$

Divide both sides by 2. $y \geq \frac{3}{2}x - 3$

$m = \frac{3}{2} = \frac{\text{change in } y}{\text{change in } x}$

$b = {}^{-}3$. The y-intercept is at the point $(0,{}^{-}3)$.

The slope tells you to go up 3 spaces and right 2 for (2,0).

Use a **solid** line for the border and shade **above** the line because the symbol is $\geq$.

* * * * * * * * * * * *

$y \geq 5 - \frac{5}{2}x$

Use the commutative property. $y \geq {}^{-}\frac{5}{2}x + 5$

$m = \frac{{}^{-}5}{2} = \frac{\text{change in } y}{\text{change in } x}$

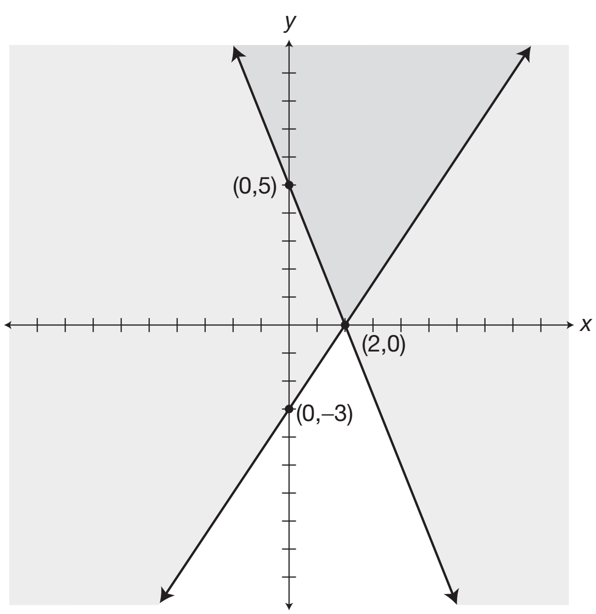
{Use the negatives to keep the coordinates near the origin.}

$b = 5$. The y-intercept is at the point (0,5).

The slope tells you to go down 5 spaces and right 2 for (2,0).

Use a **solid** line for the border and shade **above** the line because the symbol is $\geq$.

<u>The solution for the system of inequalities is where the shaded areas overlap.</u>



268. Transform equations into slope/y-intercept form. $y - x \geq 6$

Add x to both sides. $y - x + x \geq x + 6$

Simplify. $y \geq x + 6$

$m = 1 = \frac{^{-}1}{^{-}1} = \frac{\text{change in } y}{\text{change in } x}$

$b = 6$. The y-intercept is at the point (0,6).

The slope tells you to go down 1 space and left 1 for ($^{-}1$,5).

Use a **solid** line for the border and shade **above** it because the symbol is ≥.

* * * * * * * * * * * *

Use the distributive property of multiplication. $11y \geq {}^{-}2(x + 11)$

Simplify. $11y \geq {}^{-}2x - 22$

Divide both sides by 11. $y \geq \frac{^{-}2}{11}x - 2$

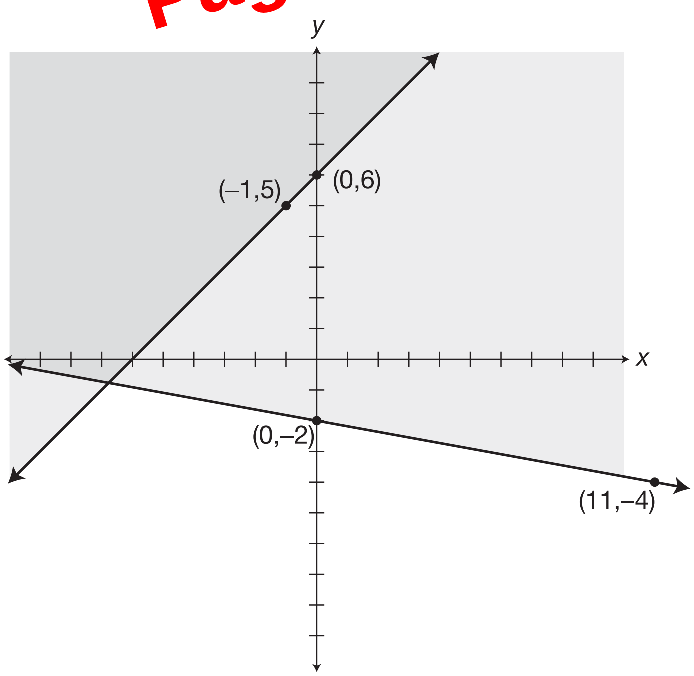
$m = \frac{^{-}2}{11} = \frac{\text{change in } y}{\text{change in } x}$

$b = {}^{-}2$. The y-intercept is at the point (0,$^{-}2$).

The slope tells you to go down 2 spaces and right 11 for (11,$^{-}4$).

Use a **solid** line for the border and shade **above** the line because the symbol is ≥.

<u>The solution for the system of inequalities is the double-shaded area on the graph.</u>



272. Transform equations into slope/y-intercept form. $9(y - 4) < 4x$

Use the distributive property of multiplication. $9y - 36 < 4x$

Add 36 to both sides. $9y - 36 + 36 < 4x + 36$

Divide both sides by 9. $y < \frac{4}{9}x + 4$

$m = \frac{4}{9} = \frac{\text{change in } y}{\text{change in } x}$

$b = 4.$ The y-intercept is at the point (0,4).

The slope tells you to go up 4 spaces and right 9 for (9,8).

Use a **dotted** line for the border and shade **below** it because the symbol is <.

* * * * * * * * * * * *

$^-9y < 2(x + 9)$

Use the distributive property of multiplication. $^-9y < 2x + 18$

Divide both sides by $^-9$. Change the direction of the symbol when dividing by a negative. $y > \frac{^-2}{9}x - 2$

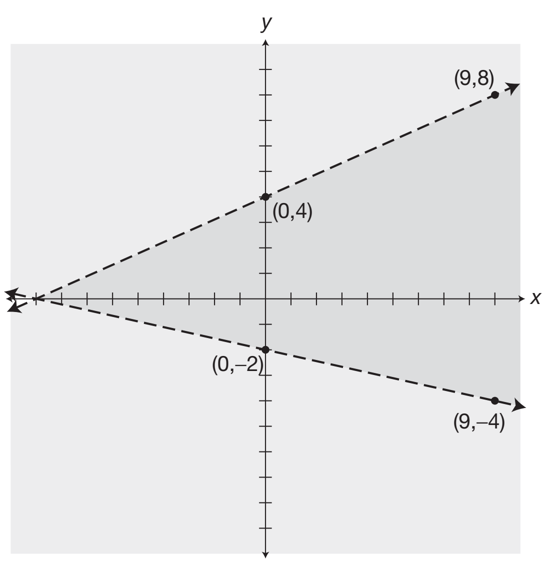
$m = \frac{^-2}{9} = \frac{\text{change in } y}{\text{change in } x}$

$b = {}^-2.$ The y-intercept is at the point $(0,{}^-2)$.

The slope tells you to go down 2 spaces and right 9 for $(9,{}^-4)$.

Use a **dotted** line for the border and shade **above** the line because the symbol is >.

The solution for the system of inequalities is the double-shaded area on the graph.



Tips for Solving Systems of Equations Algebraically

When using the elimination method, first make a plan to determine which variable you will eliminate from the system. Then transform the equation or equations so that you will get the result you want.

Express your solution as a coordinate point or in the form (x,y), or as variables such as $x = 2$ and $y = 4$.

Use the elimination method to solve the following systems of equations.

276. $x + y = 4$
$2x - y = {}^-1$

277. $3x + 4y = 17$
${}^-x + 2y = 1$

278. $7x + 3y = 11$
$2x + y = 3$

279. $0.5x + 5y = 28$
$3x - y = 13$

280. $3(x + y) = 18$
$5x + y = {}^-2$

281. $\frac{1}{2}x + 2y = 11$
$2x - y = 17$

282. $5x + 8y = 25$
$3x - 15 = y$

283. $6y + 3x = 30$
$2y + 6x = 0$

284. $3x = 5 - 7y$
$2y = x - 6$

285. $3x + y = 20$
$\frac{x}{3} + 10 = y$

286. $2x + 7y = 45$
$3x + 4y = 22$

287. $3x - 5y = {}^-21$
$2(2y - x) = 16$

288. $\frac{1}{4}x + y = 12$
$2x - \frac{1}{3}y = 21$

Use the substitution method to solve the following systems of equations.

289. $y = 5x$
$4x + 5y = 87$

290. $x + y = 3$
$3x + 101 = 7y$

291. $5x + y = 3.6$
$y + 21x = 8.4$

292. $8x - y = 0$
$10x + y = 27$

293. $\frac{x}{3} = y + 2$
$2x - 4y = 32$

294. $y + 3x = 0$
$y - 3x = 24$

295. $5x + y = 20$
$3x = \frac{1}{2}y + 1$

296. $2x + y = 2 - 5y$
$x - y = 5$

297. $\frac{2x}{10} + \frac{y}{5} = 1$
$3x + 2y = 12$

298. $x + 6y = 11$
$x - 3 = 2y$

299. $4y + 31 = 3x$
$y + 10 = 3x$

300. $2(2 - x) = 3y - 2$
$3x + 9 = 4(5 - y)$

Substitute the value of x into one of the equations in the system and solve for y.	$5(^-2) + y = {}^-2$
	$^-10 + y = {}^-2$
Add 10 to both sides.	$10 - 10 + y = 10 - 2$
Simplify.	$y = 8$

<u>The solution for the system of equations is ($^-2$,8).</u>

281.

Multiply the second equation by 2 and add to the first to eliminate y.	$2(2x - y) = 2(17)$
Use the distributive property of multiplication.	$2(2x) - 2(y) = 2(17)$
Simplify.	$4x - 2y = 34$
Add the first equation to the second.	$\underline{\frac{1}{2}x + 2y = 11}$
	$4\frac{1}{2}x + 0 = 45$
Additive identity.	$4\frac{1}{2}x = 45$
Multiply the equation by 2 to simplify the fraction.	$2(4\frac{1}{2}x = 45)$
Simplify.	$9x = 90$
Divide both sides by 9.	$x = 10$
Substitute the value of x into one of the equations in the system and solve for y.	$2(10) - y = 17$
Subtract 20 from both sides.	$20 - 20 - y = 17 - 20$
Combine like terms on each side.	$^-y = {}^-3$
Multiply the equation by $^-1$.	$y = 3$

<u>The solution for the system of equations is (10,3).</u>

282.

Transform the second equation into a similar format to the first equation, then line up like terms.	
Add 15 to both sides.	$3x - 15 + 15 = y + 15$
Simplify.	$3x = y + 15$
Subtract y from both sides.	$3x - y = y - y + 15$
Simplify.	$3x - y = 15$
Multiply the second equation by 8 and add the first equation to the second.	$8(3x - y) = 15$
Use the distributive property of multiplication.	$24x - 8y = 120$
Add the first equation to the second.	$\underline{5x + 8y = 25}$
	$29x + 0 = 145$
Additive identity.	$29x = 145$
Divide both sides by 29.	$x = 5$
Substitute the value of x into one of the equations in the system and solve for y.	$3(5) - 15 = y$
Simplify.	$15 - 15 = y$
	$0 = y$

<u>The solution for the system of equations is (5,0).</u>

289. The first equation tells you that $y = 5x$.
Substitute $5x$ for y in the second equation and then solve for x. $4x + 5(5x) = 87$
Simplify term and add like terms. $4x + 25x = 87$
$29x = 87$
Divide both sides by 29. $x = \frac{87}{29} = 3$
Substitute 3 for x in one of the equations. $y = 5 \cdot (3) = 15$
The solution for the system of equations is (3,15).

290. Transform the first equation so that the value of x is expressed in terms of y.
Subtract y from both sides of the equation. $x + y - y = 3 - y$
Simplify. $x = 3 - y$
Substitute $3 - y$ for x in the second equation and solve for y. $3(3 - y) + 101 = 7y$
Use the distributive property of multiplication. $9 - 3y + 101 = 7y$
Use the commutative property of addition. $9 + 101 - 3y = 7y$
Add like terms. Add $3y$ to both sides. $110 - 3y + 3y = 7y + 3y$
Combine like terms. $110 = 10y$
Divide both sides by 10. $11 = y$
Substitute the value of y into one of the equations in the system and solve for x. $x + (11) = 3$
Subtract 11 from both sides. $x + 11 - 11 = 3 - 11$
Combine like terms on each side. $x = {}^{-}8$
The solution for the system of equations is $({}^{-}8,11)$.

291. Transform the first equation so that y is expressed in terms of x. $5x + y = 3.6$
Subtract $5x$ from both sides of the equation. $5x - 5x + y = 3.6 - 5x$
Combine like terms on each side. $y = 3.6 - 5x$
Substitute the value of y into the second equation. $(3.6 - 5x) + 21x = 8.4$
Combine like terms. $3.6 + 16x = 8.4$
Subtract 3.6 from both sides. $3.6 - 3.6 + 16x = 8.4 - 3.6$
Combine like terms on each side. $16x = 4.8$
Divide both sides by 16. $x = 0.3$
Substitute the value of x into one of the equations in the system and solve for y. $5(0.3) + y = 3.6$
Simplify terms. $1.5 + y = 3.6$
Subtract 1.5 from both sides. $1.5 - 1.5 + y = 3.6 - 1.5$
Combine like terms on each side. $y = 2.1$
The solution for the system of equations is (0.3,2.1).

When dividing variables with exponents, if the variables are the same, you subtract the exponents:

$$\frac{n^5}{n^2} = \frac{n \cdot n \cdot n \cdot n \cdot n}{n \cdot n} = n^{5-2} = n^3$$

If the exponent of a similar term in the denominator is larger than the one in the numerator, the exponent will have a negative sign:

$$\frac{2x^3}{x^4} = 2x^{-1}$$

$$\frac{n^5}{n^8} = n^{5-8} = n^{-3}$$

A negative numerator becomes positive when the variable is moved into the denominator.

$$2x^{-1} = 2\left(\frac{1}{x^1}\right) = \frac{2}{x}$$

$$n^{-3} = \frac{1}{n^3}$$

When the result of a division leaves an exponent of zero, the term raised to the power of zero equals 1:

$$z^0 = 1$$

$$3\frac{r^2}{r^2} = 3r^0 = 3(1) = 3$$

When a variable with an exponent is raised to a power, you multiply the exponent to form the new term:

$$(b^2)^3 = b^2 \cdot b^2 \cdot b^2 = b^{(2+2+2)} = b^6$$

$$(2x^2y)^2 = 2x^2y \cdot 2x^2y = 2 \cdot 2 \cdot x^2 \cdot x^2 \cdot y \cdot y = 2^2x^{2+2}y^{1+1} = 4x^4y^2$$

Remember order of operations: PEMDAS. Generally, list terms in order from highest power to lowest power.

337. Use **FOIL** to multiply binomials.

Multiply the **first** terms in each binomial. $([28x] + 7)([\frac{x}{7}] - 11)$
$4x^2$

Multiply the **outer** terms in each binomial. $([28x] + 7)(\frac{x}{7} - [11])$
$^{-}308x$

Multiply the **inner** terms in each binomial. $(28x + [7])([\frac{x}{7}] - 11)$
^{+}x

Multiply the **last** terms in each binomial. $(28x + [7])(\frac{x}{7} - [11])$
$^{-}77$

Add the products of FOIL together. $4x^2 - 308x + x - 77$

Combine like terms. $\underline{4x^2 - 307x - 77}$

338. Use **FOIL** to multiply binomials.

Multiply the **first** terms in each binomial. $([3x^2] + y^2)([x^2] - 2y^2)$
$3x^4$

Multiply the **outer** terms in each binomial. $([3x^2] + y^2)(x^2 - [2y^2])$
$^{-}6x^2y^2$

Multiply the **inner** terms in each binomial. $(3x^2 + [y^2])([x^2] - 2y^2)$
$^{+}x^2y^2$

Multiply the **last** terms in each binomial. $(3x^2 + [y^2])(x^2 - [2y^2])$
$^{-}2y^4$

Add the products of FOIL together. $3x^4 - 6x^2y^2 + x^2y^2 - 2y^4$

Combine like terms. $\underline{3x^4 - 5x^2y^2 - 2y^4}$

339. Use **FOIL** to multiply binomials.

Multiply the **first** terms in each binomial. $([4] + 2x^2)([9] - 3x)$
$^{+}36$

Multiply the **outer** terms in each binomial. $([4] + 2x^2)(9 - [3x])$
$^{-}12x$

Multiply the **inner** terms in each binomial. $(4 + [2x^2])([9] - 3x)$
$^{+}18x^2$

Multiply the **last** terms in each binomial. $(4 + [2x^2])(9 - [3x])$
$^{-}3x^3$

Add the products of FOIL together. $36 - 12x + 18x^2 - 3x^3$

Simplify and put them in order from the highest power. $\underline{^{-}3x^3 + 18x^2 - 12x + 36}$

340. Use **FOIL** to multiply binomials.

Multiply the **first** terms in each binomial. $([2x^2] + y^2)([x^2] - y^2)$
$2x^4$

Multiply the **outer** terms in each binomial. $([2x^2] + y^2)(x^2 - [y^2])$
$^{-}2x^2y^2$

Multiply the **inner** terms in each binomial. $(2x^2 + [y^2])([x^2] - y^2)$
$^{+}x^2y^2$

Multiply the **last** terms in each binomial. $(2x^2 + [y^2])(x^2 - [y^2])$
$^{-}y^4$

Add the products of FOIL together. $2x^4 - 2x^2y^2 + x^2y^2 - y^4$

Combine like terms. $\underline{2x^4 - x^2y^2 - y^4}$

factored form: ([ ] ± [ ])([ ] ± [ ]). The factors of the first term go in the first position in the parentheses and the factors of the third term go in the second position in each factor, e.g. $x^2 + 2x + 1 = (x + 1)(x + 1)$.

Factor the following polynomials.

351. $9a + 15$

352. $3a^2x + 9ax$

353. $x^2 - 16$

354. $4a^2 - 25$

355. $7n^2 - 28n$

356. $7x^4y^2 - 35x^2y^2 + 14x^2y^4$

357. $x^2 + 3x + 2$

358. $9r^2 - 49$

359. $x^2 - 2x - 8$

360. $x^2 + 5x + 6$

361. $x^2 + x - 6$

362. $b^2 - 100$

363. $x^2 + 7x + 12$

364. $x^2 - 3x - 18$

365. $b^2 - 6b + 8$

366. $b^2 - 4b - 21$

367. $a^2 + 11a - 12$

368. $x^2 + 10x + 25$

369. $36y^4 - 4z^2$

370. $x^2 + 20x + 99$

371. $c^2 - 12c + 32$

372. $h^2 - 12h + 11$

373. $m^2 - 11m + 18$

374. $v^4 - 13v^2 - 48$

375. $x^2 - 20x + 36$

372. The factors of h^2 are h and h, and the factors of 11 are (1)(11). The sign of the numerical term is positive, so the signs in the factors of our trinomial factorization must be the same. The sign of the first-degree term (the variable to the power of 1) is negative. So use negative signs in the trinomial factors. Check your answer. $\underline{(h - 1)(h - 11) = h^2 - 11h - 1h + 11 = h^2 - 12h + 11}$

373. This expression can be factored using the trinomial method. The factors of m^2 are m and m, and the factors of 18 are (1)(18) or (2)(9) or (3)(6). The sign of the numerical term is positive, so the signs in the factors of our trinomial factorization must be the same. The sum of the results of multiplying the Outer and Inner terms of the trinomial factors needs to add up to $^{-}11m$. The only factors of 18 that can be added or subtracted in any way to equal 11 are 2 and 9. Use them and two subtraction signs in the trinomial factor terms. Check your answer using FOIL. $\underline{(m - 2)(m - 9) = m^2 - 9m - 2m + 18 = m^2 - 11m + 18}$

374. This expression can be factored using the trinomial method. The factors of v^4 are $(v^2)(v^2)$, and the factors of 48 are (1)(48) or (2)(24) or (3)(16) or (4)(12) or (6)(8). Only the product of a positive and a negative numerical term will result in $^{-}48$. The only factors of 48 that can be added or subtracted in any way to equal 13 are 3 and 16. Use 3 and 16 and a positive and negative sign in the terms of the trinomial factors. Check your answer using FOIL. $(v^2 + 3)(v^2 - 16) = v^4 - 16v^2 + 3v^2 - 48 = v^4 - 13v - 48$

You may notice that one of the two factors of the trinomial expression can itself be factored. The second term is the difference of two perfect squares. Factor $(v^2 - 16)$ using the form for factoring the difference of two perfect squares. $(v + 4)(v - 4) = v^2 - 4v + 4v - 16 = v^2 - 16$

This now makes the complete factorization of $\underline{v^4 - 13v^2 - 48 = (v^2 + 3)(v + 4)(v - 4).}$

Our choices are $(5)(1)c + (1)(^-2)c$, $(5)(^-1)c + (1)(2)$, $(5)(2)c + (1)(^-1)c$, $(5)(^-2)c + (1)(1)c$. The last of these is equal to the desired ^-9c, which gives the factoring $(5c + 1)(c - 2)$. Check using FOIL to multiply terms. $(5c + 1)(c - 2)$

Check using FOIL.

First—$^-(5c)(c) = 5c^2$

Outer—$^-(5c)(^-2) = {}^-10c$

Inner—$^-(1)(c) = c$

Last—$^-(1)(^-2) = {}^-2$

The product of the factors $\underline{(5c + 1)(c - 2) = 5c^2 - 10c + c - 2 = 5c^2 - 9c - 2}$.

392. The terms of the expression have a greatest common factor of x. Factoring x out of the expression results in $x(9x^2 - 4)$.

The expression inside the parentheses is the difference of two perfect squares. Factor that expression using the form for the difference of two perfect squares. Include the greatest common factor to complete the factorization of the original expression.

$9x^2 = (3x)^2$

$4 = 2^2$

Using the form, the factorization of the difference of two perfect squares is $(3x - 2)(3x + 2)$.

Check using FOIL.

First—$^-(3x)(3x) = 9x^2$

Outer—$^-(3x)(2) = 6x$

Inner—$^-(^-2)(3x) = {}^-6x$

Last—$^-(^-2)(2) = {}^-4$

Include the greatest common factor x in the complete factorization.

$\underline{x(3x - 2)(3x + 2) = x(9x^2 - 4) = 9x^3 - 4x}$

393. The terms in the trinomial expression are all positive, so the signs in the trinomial factor form will be positive. $(ax + (\ ))(bx + (\ ))$

The factors of the second-degree term are $8r^2 = (r)(8r)$ or $(2r)(4r)$.

The numerical term 63 has the factors (1)(63) or (3)(21) or (7)(9).

You need two sets of factors that when multiplied and added will result in a 46. Let's look at the possibilities using the $2r$ and $4r$. $2r(1) + 4r(63) = 254r$ is too much. $2r(3) + 4r(21) = 87r$ is still too much. Try $2r(21)$ and $4r(3) = 54r$. Getting closer. $2r(7) + 4r(9) = 50r$. Nope. Now try $2r(9) + 4r(7) = 46r$. Bingo!

$(2r + 7)(4r + 9)$

Check using FOIL.

First—$^-(2r)(4r) = 8r^2$

Outer—$^-(2r)(9) = 18r$

Inner—$^-(4r)(7) = 28r$

Last—$^-(7)(9) = 63$

Add the result of the multiplication. $8r^2 + 18r + 28r + 63 = 8r^2 + 46r + 63$

The factors check out. $\underline{(2r + 7)(4r + 9) = 8r^2 + 46r + 63}$

394. When you think of $x^4 = (x^2)^2$, you can see that the expression is a trinomial that is easy to factor.

The numerical term is positive, so the signs in the trinomial factor form will be the same. The sign of the first-degree term is negative, so you will use two – signs. $(ax - (\ ))(bx - (\ ))$

The factors of the second-degree term are $4x^4 = (x^2)(4x^2)$ or $(2x^2)(2x^2)$.

The numerical term 9 has (1)(9) or (3)(3) as factors.

What combination will result in a total of 37 when the Outer and Inner products are determined? $4x^2(9) = 36x^2$, $1x^2\ (1) = 1x^2$ and $36x^2 + 1x^2 = 37x^2$. Use these factors in the trinomial factor form.

$(4x^2 - 1)(x^2 - 9)$

Check using FOIL and you will find

$(4x^2 - 1)(x^2 - 9) = 4x^4 - 36x^2 - x^2 + 9 = 4x^4 - 37x^2 + 9$.

Now you need to notice that the factors of the original trinomial expression are both factorable. Why? Because they are both the difference of two perfect squares.

Use the factor form for the difference of two perfect squares for each factor of the trinomial.

$(4x^2 - 1) = (2x + 1)(2x - 1)$

$(x^2 - 9) = (x + 3)(x - 3)$

Put the factors together to complete the factorization of the original expression.

$\underline{(4x^2 - 1)(x^2 - 9) = (2x + 1)(2x - 1)(x + 3)(x - 3)}$

395. The negative sign in front of the numerical term tells you that the signs of the trinomial factors will be + and –. $(ax + (\ ))(bx - (\ ))$

This expression has a nice balance to it with 12 at the extremities and a modest 7 in the middle. Let's guess at some middle of the road factors to plug in. Use FOIL to check.

$(4d + 3)(3d - 4)$

Using FOIL, you find

$(4d + 3)(3d - 4) = 12d^2 - 16d + 9d - 12 = 12d^2 - 7d - 12$.

Those are the right terms but the wrong signs. Try changing the signs around.

$(4d - 3)(3d + 4)$

Multiply the factors using FOIL.

$\underline{(4d - 3)(3d + 4) = 12d^2 + 16d - 9d - 12 = 12d^2 + 7d - 12}$

This is the correct factorization of the original expression.

396. Each term in the expression has a common factor of $2xy$. When factored out, the expression becomes $2xy(2y^2 + 3y - 5)$. Now factor the trinomial in the parentheses.

The last sign is negative, so the signs within the factor form will be a + and –.

$(ax + (\ ))(bx - (\ ))$

The factors of the second-degree term are $2y^2 = y(2y)$.
The numerical term 5 has factors (5)(1).
Place the factors of the second degree and the numerical terms so that the result of the Outer and Inner multiplication of terms within the factor form of a trinomial expression results in a ^+3x.
$(2y + 5)(y - 1)$
Multiply using FOIL. $(2y + 5)(y - 1) = 2y^2 - 2y + 5x - 5 = 2y^2 + 3x - 5$
The factors of the trinomial expression are correct. Now include the greatest common factor to complete the factorization of the original expression.
$2xy(2y + 5)(y - 1) = 2xy(2y^2 - 2y + 5x - 5) = 2xy(2y^2 + 3x - 5)$

397. The terms of the trinomial have a greatest common factor of $2a$. When factored out, the resulting expression is $2a(2x^2 - 19x - 33)$. The expression within the parentheses is a trinomial and can be factored.
The signs within the terms of the factor form will be + and – because the numerical term has a negative sign. Only a (+)(–) = (–). $(ax + (\ ))(bx - (\ ))$
The factors of the second-degree term are $2a^2 = a(2a)$.
The numerical term 33 has (1)(33) or (3)(11) as factors.
Since $2a(11) = 22a$, and $a(3) = 3a$, and $22a - 3a = 19a$. Use those factors in the trinomial factor form so that the result of the multiplication of the Outer and Inner terms results in $19x$.
$(2x + 3)(x - 11)$
Check using FOIL. $(2x + 3)(x - 11) = 2x^2 - 22x + 3x - 33 = 2x^2 - 19x - 33$
The factorization of the trinomial factor is correct. Now include the greatest common factor of the original expression to get the complete factorization of the original expression.
$2a(2x + 3)(x - 11) = 2a(2x^2 - 22x + 3x - 33) = 2a(2x^2 - 19x - 33)$

398. The signs within the terms of the factor form will be + and – because the numerical term has a negative sign. $(ax + (\ ))(bx - (\ ))$
The factors of the second-degree term are $3c^2 = c(3c)$.
The numerical term 40 has (1)(40) or (2)(20) or (4)(10) or (5)(8) as factors.
You want the result of multiplying and then adding the Outer and Inner terms of the trinomial factor form to result in a ^+19c when the like terms are combined. Using trial and error, you can determine that $3c(8) = 24c$, and $c(5) = 5c$, and $24c - 5c = 19c$. Use those factors in the factor form in such a way that you get the result you seek.
$(3c - 5)(c + 8) = 3c^2 + 24c - 5c - 40 = 3c^2 + 19c - 40$
The complete factorization of the original expression is $(3c - 5)(c + 8)$.

18

Simplifying Radicals

This chapter will give you practice in operating with radicals. You will not always be able to factor polynomials by factoring whole numbers and whole number coefficients. Nor do all trinomials with whole numbers have whole numbers for solutions. In these last chapters, you will need to know how to operate with radicals.

The radical sign $\sqrt{\ }$ tells you to find the root of a number. The number under the radical sign is called the *radicand*. Generally, a number has two roots, one positive and one negative. It is understood in mathematics that $\sqrt{\ }$ or $^{+}\sqrt{\ }$ is telling you to find the positive root. The symbol $^{-}\sqrt{\ }$ tells you to find the negative root. The symbol $^{\pm}\sqrt{\ }$ asks for both roots.

Tips for Simplifying Radicals

Simplify radicals by completely factoring the radicand and taking out the square root. The most thorough method for factoring is to do a prime factorization of the radicand. Then you look for square roots that can be factored out of the radicand.

458. Subtract 3 from both sides of the equation isolating the radical. $\sqrt{5x-4} = 9$

Square both sides of the equation. $(\sqrt{5x-4})^2 = 9^2$

Simplify terms on both sides. $5x - 4 = 81$

Add 4 to both sides and then divide by 5. $x = \frac{85}{5} = 17$

Check your solution in the original equation. $\sqrt{5(17)-4} + 3 = 12$

Simplify terms under the radical sign. $\sqrt{81} + 3 = 12$

Find the positive square root of 81. $9 + 3 = 12$

Simplify. $12 = 12$

The solution $x = 17$ checks out.

459. Square both sides of the equation. $(\sqrt{4x+9})^2 = (-13)^2$

Simplify terms on both sides of the equation. $4x + 9 = 169$

Subtract 9 from both sides and then divide by 4. $x = 40$

Substitute the solution in the original equation. $\sqrt{4(40+9)} = -13$

Simplify the expression under the radical sign. $\sqrt{169} = -13$

The radical sign calls for the positive square root. $13 \neq -13$

The solution does not check out.

There is no solution for this equation.

460. Subtract 3 from both sides isolating the radical. $\sqrt{5x-6} = 8$

Square both sides of the equation. $(\sqrt{5x-6})^2 = 8^2$

Simplify terms. $5x - 6 = 64$

Add 6 to both sides and divide the result by 5. $x = 14$

Check the solution in the original equation. $\sqrt{5(14)-6} + 3 = 11$

Simplify the expression under the radical. $\sqrt{64} + 3 = 11$

Find the positive square root of 64 and add 3. $8 + 3 = 11$

The solution $x = 14$ checks out. $11 = 11$

461. Subtract 14 from both sides to isolate the radical. $\sqrt{9-x} = 11$

Now square both sides of the equation. $9 - x = 121$

Subtract 9 from both sides. $-x = 112$

Multiply both sides by negative 1 to solve for x. $x = -112$

Check the solution in the original equation. $\sqrt{9-(-112)} + 14 = 25$

Simplify the expression under the radical sign. $\sqrt{121} + 14 = 25$

The square root of 121 is 11. Add 14 and the solution $x = -112$ checks. $25 = 25$

factoring quadratic equations.) It will be important to check each solution. $x^2 + 2x - 8 = (x + 4)(x - 2) = 0$

Let the first factor equal zero and solve for x. $x + 4 = 0$

Subtract 4 from both sides. $x = {}^-4$

Check the solution in the original equation. $({}^-4) = \sqrt{8 - 2({}^-4)}$

Evaluate the expression under the radical sign. ${}^-4 = \sqrt{16}$

The radical sign calls for a positive root. ${}^-4 \neq 4$

Therefore, x cannot equal ${}^-4$. $x = {}^-4$ is an example of an *extraneous root.*

Let the second factor equal zero and solve for x. $x - 2 = 0$

Subtract 2 from both sides. $x = 2$

Check the solution in the original equation. $(2) = \sqrt{8 - 2(2)}$

Evaluate the expression under the radical sign. $2 = \sqrt{4}$

The positive square root of 4 is 2. $2 = 2$

Therefore, the only solution for the equation is $\underline{x = 2}$.

470. With the radical alone on one side of the equation, square both sides. $x^2 = 3x + 4$

The resulting quadratic equation may have up to two solutions. Put it into standard form and factor the equation using the trinomial factor form to find the solutions. Then check the solutions in the original equation. $x^2 - 3x - 4 = (x - 4)(x + 1) = 0$

Letting each factor equal zero and solving for x results in two possible solutions, $x = 4$ and/or ${}^-1$. Check the first possible solution in the original equation. $(4) = \sqrt{3(4) + 4} = \sqrt{16} = 4$

The solution checks out. Now check the second possible solution in the original equation. $({}^-1) = \sqrt{3({}^-1) + 4} = \sqrt{1} = 1$

${}^-1 \neq 1$

Therefore, $x \neq {}^-1$. $x = {}^-1$ is an *extraneous root*.

The only solution for the original equation is $\underline{x = 4}$.