5.6 RULES OF DIFFERENTIATION

Many functions encountered in applications of mathematics to practical problems are composed of combinations of elementary functions such as x^n and $\sin x$. It is convenient to have available a set of rules that enable differentiation to be carried out in a straightforward manner for such functions. In the following cases it is assumed that y is the dependent variable, x the independent variable, and u, v and w are variables such that

$$y = f(x), \quad u = u(x), \quad v = v(x), \quad w = w(x) .$$

Rule 1: If y may be expressed as a constant multiple of the variable u so that

$$y = f(x) = cu = cu(x) ,$$

where c is a constant then

$$y + \Delta y = f(x + \Delta x) = c\{u + \Delta u\} = cu(x + \Delta x) ,$$

where Δu is the change in u due to the change Δx in x. Then

$$\frac{\Delta y}{\Delta x} = c\frac{\Delta u}{\Delta x}$$

so that in the limit

$$\frac{dy}{dx} = c\frac{du}{dx} \qquad (5.9)$$

or

$$f'(x) = cu'(x) . \qquad (5.10)$$

Rules 2, 3 and 4 are proved by methods similar to those used to prove rule 1, and they will be stated here. Proofs may be found in any standard book on calculus such as Smyrl (1978).

Rule 2: If y may be expressed as the sum (or difference) of the two variables u and v so that

$$y = f(x) = u \pm v = u(x) \pm v(x)$$

then

$$\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx} \qquad (5.11)$$

or

$$f'(x) = u'(x) \pm v'(x) . \qquad (5.12)$$

This rule may be extended to cover the case

$$y = u + v + w$$

when

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} .$$