14. Insert 5 arithmetic means between -1 and 23.
   A. 3, 7, 11, 15, 19
   B. 7, 10, 13, 16, 19
   C. -1, 3, 7, 11, 15
   D. 1, 5, 9, 13, 17

15. An object dropped from a cliff will fall 16 feet the first second, 48 feet the second, 80 feet the third, and so on, increasing by 32 feet each second. What does the total distance the object will fall in 7 seconds?
   A. 874 ft.
   B. 847 ft.
   C. 748 ft.
   D. 784 ft.

16. A besieged fortress is held by 5700 men who have provisions for 66 days. If the garrison loses 20 men each day, how many days can the provision hold out?
   A. 70 days
   B. 76 days
   C. 67 days
   D. 80 days

17. In a racing contest, there are 240 cars with fuel provision for 15 hours each. Assuming a constantly hourly consumption for each car, how long will the fuel provision last if 8 cars withdraw from the race every hour?
   A. 72 hours
   B. 23 hours
   C. 25 hours
   D. 20 hours

18. How many numbers divisible by 4 lie between 70 and 203?
   A. 33
   B. 35
   C. 34
   D. 36

19. Find the sum of the numbers divisible by between 70 and 203.
   A. 4848
   B. 4488
   C. 8484
   D. 8844

20. Two men set out from a certain place going in the same direction. The first travels at a constant rate of 8 kilometers per hour, while the second goes 4 km for the first hour, 4.5 km the second hour, 5 km the third hour, and so on. After how many hours will the second man overtake the first?
   A. 18 hours
   B. 20 hours
   C. 17 hours
   D. 19 hours

21. Ten balls are placed in a straight line on the ground at intervals of 2 meters. Six meters from the end of the row a basket is placed. A boy starts from the basket and picks up the balls and carries them, one at a time, to the basket. How far did he walk all in all?
   A. 120 m
   B. 130 m
   C. 250 m
   D. 260 m

22. Find the third term of a geometric sequence whose first term is 2 and whose fifth term is 18.
   A. 12
   B. 6
   C. 9
   D. 27

23. A basketball is dropped from a height of 10m. On each rebound it rises 2/3 of the height from which it last fell. Determine the total distance travelled until it comes to rest.
   A. 45 m
   B. 60 m
   C. 50 m
   D. 75 m

24. Find the sum of the geometric progression 2, 6, 18, ... up to the 10th term.
   A. 10,682
   B. 177,146
   C. 59,048
   D. 6,560

ALGEBRA: SEQUENCE AND SERIES SOLUTIONS

1. Find the n^{th} term of the arithmetic progression 6, 10, 14, ...
   First Solution:
   Using the formula:
   $$a_n = a_1 + (n-1)d$$
   $a_1 = 6 \rightarrow$ first term
   $d = 4 \rightarrow$ common difference
   We have:
   $$a_n = 6 + (n-1)(4)$$
   $$a_n = 6 + 4n - 4$$
   $$a_n = 2 + 4n$$

   Second Solution:
   Since the relationship between a_n and n is linear, hence we can use here the STAT Mode 3-2 → A + Bx
   For Linear Mathematical Model, we only need 2 points to define the function in the form of $y = A + Bx$.
   Input:

x	y
1	6
2	10

   Press AC.
   Then press Shift 1 – 5 (Reg – Regression)
   Then select 1: A and then select 2: B
   $A = 2$; $B = 4$
   Therefore, $y = A + Bx$
   $$a_n = 2 + 4n$$

2. Find the sum up to the 10^{th} term of the arithmetic progression 6, 10, 14, ...
   First Solution:
   By using the formula of the sum of an arithmetic progression of n terms:
   $$s_n = \frac{[2a_1 + (n-1)d]n}{2}$$
   $$s_n = \frac{[2(6) + (10-1)(4)](10)}{2}$$
   $$s_n = 240$$

   Second Solution:
   From the formula, it shows that the relationship between n and s_n is in quadratic form, so we can use STAT MODE 3-3 → $A + Bx + Cx^2$
   For Quadratic Mathematical Model, we need 3 points to define the function in the form of $y = A + Bx + Cx^2$.
   Input:

x	y
1	6
2	6 +10 = 16
3	6 + 10 +14 = 30

   For the sum of the first 10 terms:
   Find the value of y which corresponds to the value of $x = 10$.
   Hence $10\hat{y} = 240$.

Situation: The distance a ball rolls down a ramp each second is given by the arithmetic sequence whose n^{th} term $2n - 1$ in feet.

7. Find the distance the ball rolls during the 10^{th} second.
   First Solution:
   Let: a_n be the distance traveled by ball in a certain interval at any n^{th} second
   $n = n^{th}$ second
   The distance traveled in the first second is $2n - 1$; hence $a_1 = 2(1) - 1 = 1$ foot; $a_2 = 2(2) - 1 = 3$ feet; $d = 3 - 1 = 2$ feet.

   Therefore, using the formula:
   $$a_n = a_1 + (n - 1)d$$
   We have
   $$a_{10} = 1 + (10 - 1)(2)$$
   $$a_{10} = 19 \text{ feet}$$

   Second Solution:
   Go to MODE 3-2:
   Input:

x	y
1	1
2	3

   Press AC.
   Then press $10\hat{y} = 19$ feet.

8. Find the total distance the ball travels in 10 seconds.
   First Solution:
   By using the formula of the sum of an arithmetic progression of n terms:
   $$s_n = \frac{[2a_1 + (n - 1)d]n}{2}$$
   $$s_n = \frac{[2(1) + (10 - 1)(2)](10)}{2}$$
   $$s_n = 100 \text{ feet}$$

   Second Solution:
   From the formula shows that the relationship between n and s_n is in quadratic form, we can use STAT MODE 3 → 2: $A + Bx + Cx^2$
   For Quadratic Mathematical Models we need 3 points to define the function in the form of $y = A + Bx + Cx^2$.
   Input:

x	y
1	1
2	$1 + 3 = 4$
3	$1 + 3 + 5 = 9$

   For the sum of the first 10 terms:
   Find the value of y which corresponds to the value of $x = 10$.
   Hence $10\hat{y} = 100$ feet.

Mixture Problems

21. A car radiator has a 6-liter capacity. If the liquid in the radiator is 40% antifreeze, how much liquid must be replaced with pure antifreeze to bring the mixture up to a 50% solution?

Solution:

Let: x = amount of pure antifreeze to be substituted

Amount of antifreeze in the original solution = $6(0.4) = 2.4$ L

The amount of antifreeze that will remain in the solution if we draw a volume equal to amount of pure antifreeze, before replacing a pure antifreeze is:

$$2.4 - 0.4x$$

And putting the pure antifreeze in the solution, hence the amount of antifreeze in the solution will become

$$2.4 - 0.4x + x(1) = 0.5(6)$$

$$\mathbf{x = 1\ liter}$$

22. An automobile engine can run on a mixture of gasoline and a substitute fuel. If gasoline costs P 3.50 per gallon and the substitute fuel costs P 2 per gallon, what percent of the mixture must be substitute fuel to bring the cost down to P 2.75 per gallon?

Solution:

Let:

x = volume of gasoline in every 1 gallon of solution

y = volume of substitute fuel in every 1 gallon of solution

Hence,

$$\begin{cases} x + y = 1 \rightarrow \text{eq. by volume} \\ 3.5x + 2y = 2.75 \rightarrow \text{eq. by total cost} \end{cases}$$

Solving simultaneously,

$$x = 0.5\ \text{gal}$$

$$y = 0.5\ \text{gal}$$

Therefore,

$$\frac{0.5\ \text{gal}}{1\ \text{gal}} = 50\%\ \text{of gasoline}$$

$$\frac{0.5\ \text{gal}}{1\ \text{gal}} = 50\%\ \text{of substitute fuel}$$

23. How many liters of water must evaporate to turn 12 liters of a 24% salt solution into a 36% solution?

Solution:

Let: x = amount of water to be evaporated

The amount of salt after evaporation will remain the same hence,

$$0.36(12 - x) = 2.88$$

$$\mathbf{x = 4\ liters}$$

24.A forester mixes gasoline and oil to make 2 gallons of mixture for his two-cycle chainsaw engine. This mixture is 32 parts gasoline and 1-part two-cycle oil. How much gasoline must be added to bring the mixture to 40 parts gasoline and 1-part oil?

Solution:

Let:

x = volume of gasoline in original solution

y = volume of oil in original solution

$$\begin{cases} x + y = 2 \\ \frac{x}{y} = \frac{32}{1} \end{cases}$$

Hence the system of equation will become

$$\begin{cases} x + y = 2 \\ x - 32y = 0 \end{cases}$$

$x = 1.939$ gal of gasoline

$\mathbf{y = 0.061}$ **gal of oil**

Let:

z = amount of gasoline to be added

$$\frac{1.939 + z}{0.061} = \frac{40}{1}$$

$\mathbf{z = 0.50}$ **gal**

25.How many ounces of pure gold that costs P850 per ounce must be mixed with 25 ounces of a gold alloy that costs P 500 per ounce to make a new alloy that costs P 725 per ounce?

Solution:

Let x = weight of pure gold to be mixed with 25 ounces of gold, in ounce

cost of pure gold in the new alloy = 850x

cost of 25 ounces of gold alloy in the new alloy = 25(500) = 12500

Hence,

$$850x + 12500 = 725(x + 25)$$

$\mathbf{x = 45}$ **ounces**

26.How many grams of pure silver must a silversmith mix with a 45% silver alloy to produce 200 grams of 50% silver alloy?

Solution:

x = mass of pure silver

y = mass of 45% silver alloy

$x + y = 200$ → equation due to total mass of alloy

$x + 0.45y = 0.5(200)$ → equation due to total amount of silver

Hence, solving simultaneously,

$\mathbf{x = 18.18}$ **grams → pure silver**

y = 181.818 grams → 45% silver