CHAPTER 1

Linear Coordinate Systems. Absolute Value. Inequalities

Linear Coordinate System

A linear coordinate system is a graphical representation of the real numbers as the points of a straight line. To each number corresponds one and only one point, and to each point corresponds one and only one number.

To set up a linear coordinate system on a given line: (1) select any point of the line as the *origin* and let that point correspond to the number 0; (2) choose a positive direction on the line and indicate that direction by an arrow; (3) choose a fixed distance as a unit of measure. If x is a positive number, find the point corresponding to x by moving a distance of x units from the origin in the positive direction. If x is negative, find the point corresponding to x by moving a distance of $-x$ units from the origin in the negative direction. (For example, if $x = -2$, then $-x = 2$ and the corresponding point lies 2 units from the origin in the negative direction.) See Fig. 1-1.

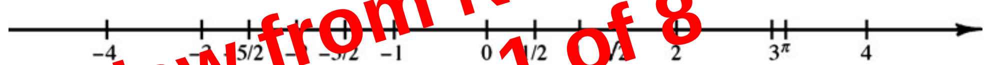

Fig. 1-1

The number assigned to a point by a coordinate system is called the *coordinate* of that point. We often will talk as if there is no distinction between a point and its coordinate. Thus, we might refer to "the point 3" rather than to "the point with coordinate 3."

The absolute value $|x|$ of a number x is defined as follows:

$$|x| = \begin{cases} x & \text{if } x \text{ is zero or a positive number} \\ -x & \text{if } x \text{ is a negative number} \end{cases}$$

For example, $|4| = 4$, $|-3| = -(-3) = 3$, and $|0| = 0$. Notice that, if x is a negative number, then $-x$ is positive. Thus, $|x| \geq 0$ for all x.

The following properties hold for any numbers x and y.

(1.1) $|-x| = |x|$
When $x = 0$, $|-x| = |-0| = |0| = |x|$.
When $x > 0$, $-x < 0$ and $|-x| = -(-x) = x = |x|$.
When $x < 0$, $-x > 0$, and $|-x| = -x = |x|$.

(1.2) $|x - y| = |y - x|$
This follows from (**1.1**), since $y - x = -(x - y)$.

(1.3) $|x| = c$ implies that $x = \pm c$.
For example, if $|x| = 2$, then $x = \pm 2$. For the proof, assume $|x| = c$.
If $x \geq 0$, $x = |x| = c$. If $x < 0$, $-x = |x| = c$; then $x = -(-x) = -c$.

(1.4) $|x|^2 = x^2$
If $x \geq 0$, $|x| = x$ and $|x|^2 = x^2$. If $x \leq 0$, $|x| = -x$ and $|x|^2 = (-x)^2 = x^2$.

(1.5) $|xy| = |x| \cdot |y|$
By (**1.4**), $|xy|^2 = (xy)^2 = x^2y^2 = |x|^2|y|^2 = (|x| \cdot |y|)^2$. Since absolute values are nonnegative, taking square roots yields $|xy| = |x| \cdot |y|$.