LECTURE 5

REAL BUSINESS CYLCE THEORY

3.5 Optimal Household Choices With Uncertainty

In this case the lagrangean is with the expectation term:

$$\mathcal{L} = E_0\left\{\sum_{t=0}^{t=\infty} \beta^t \left\{u(C_t, 1-L_t) + \lambda_{t+1}[(B_t + w_t L_t - C_t)(1+r_{t+1}) - B_{t+1}]\right\}\right\}. \quad (49)$$

The Firs Order Condition are given by:

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t E_t\left[u_{1,t} - (1+r_{t+1})\lambda_{t+1}\right] = 0 \quad (50)$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = \beta^t E_t\left[-u_{2,t} + w_t(1+r_{t+1})\lambda_{t+1}\right] = 0 \quad (51)$$

$$\frac{\partial \mathcal{L}}{\partial B_t} = \beta^t E_t[\lambda_{t+1}(1+r_{t+1}) - \beta^{t-1}\lambda_t] = 0 \quad (52)$$

which can be rewritten as:

$$u_{1,t} = E_t[(1+r_{t+1})\lambda_{t+1}]. \quad (53)$$

$$\lambda_t = \beta E_t[(1+r_{t+1})\lambda_{t+1}]. \quad (54)$$

$$u_{2,t} = w_t E_t[(1+r_{t+1})\lambda_{t+1}]. \quad (55)$$

We can derive the intertemporal optimal conditions in the same way as before. Consider, for example, the Euler equation for consumption:

$$u_{1,t} = \beta E_t\left[(1+r_{t+1})u_{1,t+1}\right] \quad (56)$$

Using again the logarithmic utility function we have:

$$\frac{1}{C_t} = \beta E_t\left[(1+r_{t+1})\frac{1}{C_{t+1}}\right]. \quad (57)$$

From this point onwards things are different with respect to the certainty case, because we have that:

$$E_t\left[(1+r_{t+1})\frac{1}{C_{t+1}}\right] = E_t(1+r_{t+1})E_t\left(\frac{1}{C_{t+1}}\right) + cov\left(1+r_{t+1}, \frac{1}{C_{t+1}}\right). \quad (58)$$

Using equation (58) in equation (57):

$$E_t\left(\frac{C_t}{C_{t+1}}\right) = \frac{1 - C_t \beta cov\left(1+r_{t+1}, \frac{1}{C_{t+1}}\right)}{\beta E_t(1+r_{t+1})}. \quad (59)$$

If $cov\left(1+r_{t+1}, \frac{1}{C_{t+1}}\right) = 0$ then we have the same result as in the certainty case: