**9.** In auction markets like the NYSE, brokers and agents meet at a physical location (the exchange) to match buyers and sellers of assets. Dealer markets like NASDAQ consist of dealers operating at dispersed locales who buy and sell assets themselves, communicating with other dealers either electronically or literally over-the-counter.

**10.** Such organizations frequently pursue social or political missions, so many different goals are conceivable. One goal that is often cited is revenue minimization; i.e., provide whatever goods and services are offered at the lowest possible cost to society. A better approach might be to observe that even a not-for-profit business has equity. Thus, one answer is that the appropriate goal is to maximize the value of the equity.

**11.** Presumably, the current stock value reflects the risk, timing, and magnitude of all future cash flows, both short-term *and* long-term. If this is correct, then the statement is false.

**12.** An argument can be made either way. At the one extreme, we could argue that in a market economy, all of these things are priced. There is thus an optimal level of, for example, ethical and/or illegal behavior, and the framework of stock valuation explicitly includes these. At the other extreme, we could argue that these are non-economic phenomena and are best handled through the political process. A classic (and highly relevant) thought question that illustrates this debate goes something like this: "A firm has estimated that the cost of improving the safety of one of its products is $30 million. However, the firm believes that improving the safety of the product will only save $20 million in product liability claims. What should the firm do?"

**13.** The goal will be the same, but the best course of action toward that goal may be different because of differing social, political, and economic institutions.

**14.** The goal of management should be to maximize the share price for the current shareholders. If management believes that it can improve the profitability of the firm so that the share price will exceed $35, then they should fight the offer from the outside company. If management believes that this bidder or other unidentified bidders will actually pay more than $35 per share to acquire the company, then they should still fight the offer. However, if the current management cannot increase the value of the firm beyond the bid price, and no other higher bids come in, then management is not acting in the interests of the shareholders by fighting the offer. Since current managers often lose their jobs when the corporation is acquired, poorly monitored managers have an incentive to fight corporate takeovers in situations such as this.

**15.** We would expect agency problems to be less severe in countries with a relatively small percentage of individual ownership. Fewer individual owners should reduce the number of diverse opinions concerning corporate goals. The high percentage of institutional ownership might lead to a higher degree of agreement between owners and managers on decisions concerning risky projects. In addition, institutions may be better able to implement effective monitoring mechanisms on managers than can individual owners, based on the institutions' deeper resources and experiences with their own management. The increase in institutional ownership of stock in the United States and the growing activism of these large shareholder groups may lead to a reduction in agency problems for U.S. corporations and a more efficient market for corporate control.

**9.** If a company raises more money from selling stock than it pays in dividends in a particular period, its cash flow to stockholders will be negative. If a company borrows more than it pays in interest, its cash flow to creditors will be negative.

**10.** The adjustments discussed were purely accounting changes; they had no cash flow or market value consequences unless the new accounting information caused stockholders to revalue the derivatives.

**11.** Enterprise value is the theoretical takeover price. In the event of a takeover, an acquirer would have to take on the company's debt, but would pocket its cash. Enterprise value differs significantly from simple market capitalization in several ways, and it may be a more accurate representation of a firm's value. In a takeover, the value of a firm's debt would need to be paid by the buyer when taking over a company. This enterprise value provides a much more accurate takeover valuation because it includes debt in its value calculation.

**12.** In general, it appears that investors prefer companies that have a steady earnings stream. If true, this encourages companies to manage earnings. Under GAAP, there are numerous choices for the way a company reports its financial statements. Although not the reason for the choices under GAAP, one outcome is the ability of a company to manage earnings, which is not an ethical decision. Even though earnings and cash flow are often related, earnings management should have little effect on cash flow (except for tax implications). If the market is "fooled" and prefers steady earnings, shareholder wealth can be increased, at least temporarily. However, given the questionable ethics of this practice, the company (and shareholders) will lose value if the practice is discovered.



**Solutions to Questions and Problems**

*NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** To find owner's equity, we must construct a balance sheet as follows:

| | | Balance Sheet | |
|---|---|---|---|
| CA | $5,100 | CL | $4,300 |
| NFA | 23,800 | LTD | 7,400 |
| | | OE | ?? |
| TA | $28,900 | TL & OE | $28,900 |

We know that total liabilities and owner's equity (TL & OE) must equal total assets of $28,900. We also know that TL & OE is equal to current liabilities plus long-term debt plus owner's equity, so owner's equity is:

OE = $28,900 – 7,400 – 4,300 = $17,200

NWC = CA – CL = $5,100 – 4,300 = $800

Now, looking at the income statement:

EBT – EBT × Tax rate = Net income

Recognize that EBT × Tax rate is simply the calculation for taxes. Solving this for EBT yields:

EBT = NI / (1– tax rate) = $6,600 / (1 – 0.35) = $10,154

Now you can calculate:

EBIT = EBT + Interest = $10,154 + 4,500 = $14,654

The last step is to use:

EBIT = Sales – Costs – Depreciation
$14,654 = $41,000 – 19,500 – Depreciation

Solving for depreciation, we find that depreciation = $6,846

**16.** The balance sheet for the company looks like this:



| Balance Sheet | | | |
|---|---|---|---|
| Cash | $195,000 | Accounts payable | $405,000 |
| Accounts receivable | 137,000 | Notes payable | 160,000 |
| Inventory | 264,000 | Current liabilities | $565,000 |
| Current assets | $596,000 | Long-term debt | 1,195,300 |
| | | Total liabilities | $1,760,300 |
| Tangible net fixed assets | 2,800,000 | | |
| Intangible net fixed assets | 780,000 | Common stock | ?? |
| | | Accumulated ret. earnings | 1,934,000 |
| Total assets | $4,176,000 | Total liab. & owners' equity | $4,176,000 |

Total liabilities and owners' equity is:

TL & OE = CL + LTD + Common stock + Retained earnings

Solving for this equation for equity gives us:

Common stock = $4,176,000 – 1,934,000 – 1,760,300 = $481,700

**17.** The market value of shareholders' equity cannot be negative. A negative market value in this case would imply that the company would pay you to own the stock. The market value of shareholders' equity can be stated as: Shareholders' equity = Max [(TA – TL), 0]. So, if TA is $8,400, equity is equal to $1,100, and if TA is $6,700, equity is equal to $0. We should note here that the book value of shareholders' equity can be negative.

To find ROE, we need to find total equity.
TL & OE = TD + TE
TE = TL & OE – TD
TE = $17,500,000 – 6,300,000 = $11,200,000

ROE = Net income / TE = 2,320,000 / $11,200,000 = .2071 or 20.71%

**3.** Receivables turnover = Sales / Receivables
Receivables turnover = $3,943,709 / $431,287 = 9.14 times

Days' sales in receivables = 365 days / Receivables turnover = 365 / 9.14 = 39.92 days

The average collection period for an outstanding accounts receivable balance was 39.92 days.

**4.** Inventory turnover = COGS / Inventory
Inventory turnover = $4,105,612 / $407,534 = 10.07 times

Days' sales in inventory = 365 days / Inventory turnover = 365 / 10.07 = 36.23 days

On average, a unit of inventory sat on the shelf 36.23 days before it was sold.

**5.** Total debt ratio = 0.63 = TD / TA

Substituting total debt plus total equity for total assets we get:

0.63 = TD / (TD + TE)

Solving this equation yields:

0.63(TE) = 0.37(TD)

Debt/equity ratio = TD / TE = 0.63 / 0.37 = 1.70

Equity multiplier = 1 + D/E = 2.70



**6.**

| | | |
|---|---|---|
| Net income | = Addition to RE + Dividends | = $430,000 + 175,000 = $605,000 |
| Earnings per share | = NI / Shares | = $605,000 / 210,000 = $2.88 per share |
| Dividends per share | = Dividends / Shares | = $175,000 / 210,000 = $0.83 per share |
| Book value per share | = TE / Shares | = $5,300,000 / 210,000 = $25.24 per share |
| Market-to-book ratio | = Share price / BVPS | = $63 / $25.24 = 2.50 times |
| P/E ratio | = Share price / EPS | = $63 / $2.88 = 21.87 times |
| Sales per share | = Sales / Shares | = $4,500,000 / 210,000 = $21.43 |
| P/S ratio | = Share price / Sales per share | = $63 / $21.43 = 2.94 times |

**17.** *a.* Current ratio = Current assets / Current liabilities
Current ratio 2008 = $68,726 / $61,434 = 1.12 times
Current ratio 2009 = $76,213 / $64,203 = 1.19 times

*b.* Quick ratio = (Current assets – Inventory) / Current liabilities
Quick ratio 2008 = ($67,726 – 38,760) / $61,434 = 0.49 times
Quick ratio 2009 = ($76,213 – 42,650) / $64,203 = 0.52 times

*c.* Cash ratio = Cash / Current liabilities
Cash ratio 2008 = $8,436 / $61,434 = 0.14 times
Cash ratio 2009 = $10,157 / $64,203 = 0.16 times

*d.* NWC ratio = NWC / Total assets
NWC ratio 2008 = ($68,726 – 61,434) / $295,432 = 2.47%
NWC ratio 2009 = ($76,213 – 64,203) / $324,519 = 3.70%

*e.* Debt-equity ratio = Total debt / Total equity
Debt-equity ratio 2008 = ($61,434 + 25,000) / $208,998 = 0.41 times
Debt-equity ratio 2009 = ($64,206 + 32,000) / $228,316 = 0.42 times

Equity multiplier = 1 + D/E
Equity multiplier 2008 = 1 + 0.41 = 1.41
Equity multiplier 2009 = 1 + 0.42 = 1.42

*f.* Total debt ratio = (Total assets – Total equity) / Total assets
Total debt ratio 2008 = ($295,432 – 208,998) / $295,432 = 0.29
Total debt ratio 2009 = ($324,519 – 228,316) / $324,519 = 0.30

Long-term debt ratio = Long-term debt / (Long-term debt + Total equity)
Long-term debt ratio 2008 = $25,000 / ($25,000 + 208,998) = 0.11
Long-term debt ratio 2009 = $32,000 / ($32,000 + 228,316) = 0.12



*Intermediate*

**18.** This is a multi-step problem involving several ratios. The ratios given are all part of the DuPont Identity. The only DuPont Identity ratio not given is the profit margin. If we know the profit margin, we can find the net income since sales are given. So, we begin with the DuPont Identity:

ROE = 0.15 = (PM)(TAT)(EM) = (PM)(S / TA)(1 + D/E)

Solving the DuPont Identity for profit margin, we get:

PM = [(ROE)(TA)] / [(1 + D/E)(S)]
PM = [(0.15)($3,105)] / [(1 + 1.4)( $5,726)] = .0339

Now that we have the profit margin, we can use this number and the given sales figure to solve for net income:

PM = .0339 = NI / S
NI = .0339($5,726) = $194.06

**7.** Apparently not! In hindsight, the firm may have underestimated costs and also underestimated the extra demand from the lower price.

**8.** Financing possibly could have been arranged if the company had taken quick enough action. Sometimes it becomes apparent that help is needed only when it is too late, again emphasizing the need for planning.

**9.** All three were important, but the lack of cash or, more generally, financial resources ultimately spelled doom. An inadequate cash resource is usually cited as the most common cause of small business failure.

**10.** Demanding cash up front, increasing prices, subcontracting production, and improving financial resources via new owners or new sources of credit are some of the options. When orders exceed capacity, price increases may be especially beneficial.

## Solutions to Questions and Problems

*NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*



*Basic*

**1.** It is important to remember that equity will not increase by the same percentage as the other assets. If every other item in the income statement and balance sheet increases by 15 percent, the pro forma income statement and balance sheet will look like this:

| Pro forma income statement | | Pro forma balance sheet | | | |
|---|---|---|---|---|---|
| Sales | $ 26,450 | Assets | $18,170 | Debt | $ 5,980 |
| Costs | 19,205 | | | Equity | 12,190 |
| Net income | $ 7,245 | Total | $18,170 | Total | $ 18,170 |

In order for the balance sheet to balance, equity must be:

Equity = Total liabilities and equity – Debt
Equity = $18,170 – 5,980
Equity = $12,190

Equity increased by:

Equity increase = $12,190 – 10,600
Equity increase = $1,590

This means that $13,600 of the new total debt is not raised externally. So, the debt raised externally, which will be the EFN is:

EFN = New total debt – (Beginning LTD + Beginning CL + Spontaneous increase in AP)
EFN = $315,044 – ($158,000 + 68,000 + 17,000 + 13,600) = $58,444

The pro forma balance sheet with the new long-term debt will be:

MOOSE TOURS INC.
Pro Forma Balance Sheet

| Assets | | Liabilities and Owners' Equity | |
|---|---|---|---|
| Current assets | | Current liabilities | |
| Cash | $ 30,360 | Accounts payable | $ 81,600 |
| Accounts receivable | 44,400 | Notes payable | 17,000 |
| Inventory | 104,280 | Total | $ 98,600 |
| Total | $ 183,480 | Long-term debt | 216,444 |
| Fixed assets | | | |
| Net plant and equipment | 495,600 | Owners' equity | |
| | | Common stock and paid-in surplus | $ 140,000 |
| | | Retained earnings | 278,632 |
| | | Total | $ 418,632 |
| Total assets | $ 697,080 | Total liabilities and owners' equity | $ 733,676 |



The funds raised by the debt issue can be put into an excess cash account to make the balance sheet balance. The excess debt will be:

Excess debt = $733,676 – 697,080 = $54,596

To make the balance sheet balance, the company will have to increase its assets. We will put this amount in an account called excess cash, which will give us the following balance sheet:

MOOSE TOURS INC.
Pro Forma Balance Sheet

| Assets | | Liabilities and Owners' Equity | |
|---|---|---|---|
| Current assets | | Current liabilities | |
| Cash | $ 30,360 | Accounts payable | $ 81,600 |
| Excess cash | 54,596 | | |
| Accounts receivable | 44,400 | Notes payable | 17,000 |
| Inventory | 104,280 | Total | $ 98,600 |
| Total | $ 238,076 | Long-term debt | 216,444 |
| Fixed assets | | | |
| Net plant and equipment | 495,600 | Owners' equity | |
| | | Common stock and paid-in surplus | $ 140,000 |
| | | Retained earnings | 278,632 |
| | | Total | $ 418,632 |
| | | Total liabilities and owners' | |
| Total assets | $ 733,676 | equity | $ 733,676 |

The excess cash has an opportunity cost that we discussed earlier. Increasing fixed assets would also not be a good idea since the company already has enough fixed assets. A likely scenario would be the repurchase of debt and equity in its current capital structure weights. The company's debt-assets and equity assets are:

Debt-assets = .7526 / (1 + .7526) = .43
Equity-assets = (1 – .7526) = .57

So, the amount of debt and equity needed will be:

Total debt needed = .43($697,080) = $291,600
Equity needed = .57($697,080) = $387,480

So, the repurchases of debt and equity will be:

Debt repurchase = ($98,600 + 216,444) – 291,600 = $23,444
Equity repurchase = $418,632 – 387,480 = $31,152

Assuming all of the debt repurchase is from long-term debt, and the equity repurchase is entirely from the retained earnings, the final pro forma balance sheet will be:

*25% Sales Growth*:

MOOSE TOURS INC.
Pro Forma Balance Sheet

| Assets | | Liabilities and Owners' Equity | |
|---|---|---|---|
| Current assets | | Current liabilities | |
| Cash | $ 31,625 | Accounts payable | $ 85,000 |
| Accounts receivable | 50,875 | Notes payable | 17,000 |
| Inventory | 108,625 | Total | $ 102,000 |
| Total | $ 191,125 | Long-term debt | $ 158,000 |
| Fixed assets | | | |
| Net plant and equipment | 516,250 | Owners' equity | |
| | | Common stock and paid-in surplus | $ 140,000 |
| | | Retained earnings | 282,886 |
| | | Total | $ 422,886 |
| | | Total liabilities and owners' | |
| Total assets | $ 707,375 | equity | $ 682,886 |

So the EFN is:

EFN = Total assets – Total liabilities and equity
EFN = $707,375 – 682,886
EFN = $24,889

**29.** The pro forma income statements for all three growth rates will be:

MOOSE TOURS INC.
Pro Forma Income Statement

| | *20% Sales Growth* | *30% Sales Growth* | *35% Sales Growth* |
|---|---|---|---|
| Sales | $1,114,800 | $1,207,700 | $1,254,150 |
| Costs | 867,600 | 939,900 | 976,050 |
| Other expenses | 22,800 | 24,700 | 25,650 |
| EBIT | $224,400 | $243,100 | $252,450 |
| Interest | 14,000 | 14,000 | 14,000 |
| Taxable income | $210,400 | $229,100 | $238,450 |
| Taxes (35%) | 73,640 | 80,185 | 83,458 |
| Net income | $136,760 | $148,915 | $154,993 |
| | | | |
| Dividends | $41,028 | $44,675 | $46,498 |
| Add to RE | 95,732 | 104,241 | 108,495 |

At a 30 percent growth rate, and assuming the payout ratio is constant, the dividends paid will be:

Dividends = ($30,810/$102,700)($135,948)
Dividends = $40,784

And the addition to retained earnings will be:

*Sales growth rate = 35% and debt/equity ratio = .75255:*

MOOSE TOURS INC.
Pro Forma Balance Sheet

| Assets | | Liabilities and Owners' Equity | |
|---|---|---|---|
| Current assets | | Current liabilities | |
| Cash | $ 34,155 | Accounts payable | $ 91,800 |
| Accounts receivable | 54,945 | Notes payable | 17,000 |
| Inventory | 117,315 | Total | $ 108,800 |
| Total | $ 206,415 | Long-term debt | $ 215,848 |
| Fixed assets | | | |
| Net plant and equipment | 557,550 | Owners' equity | |
| | | Common stock and paid-in surplus | $ 140,000 |
| | | Retained earnings | 291,395 |
| | | Total | $ 431,395 |
| | | Total liabilities and owners' | |
| Total assets | $ 763,965 | equity | $ 756,043 |

So the excess debt raised is:

Excess debt = $756,043 – 763,965
Excess debt = –$7,922

At a 35 percent growth rate, the firm will need funds in the amount of $7,922 in addition to the external debt already raised. So, the EFN will be:

EFN = $57,848 + 7,922
EFN = $65,770



**30.** We must need the ROE to calculate the sustainable growth rate. The ROE is:

ROE = (PM)(TAT)(EM)
ROE = (.067)(1 / 1.35)(1 + 0.30)
ROE = .0645 or 6.45%

Now we can use the sustainable growth rate equation to find the retention ratio as:

Sustainable growth rate = (ROE × b) / [1 – (ROE × b)]
Sustainable growth rate = .12 = [.0645(b)] / [1 – .0645(b)
b = 1.66

This implies the payout ratio is:

Payout ratio = 1 – b
Payout ratio = 1 – 1.66
Payout ratio = –0.66

We multiply this equation by:

$(TA_E / TA_E)$

Internal growth rate = $(NI / TA_E \times b) / (1 - NI / TA_E \times b) \times (TA_E / TA_E)$
Internal growth rate = $(NI \times b) / (TA_E - NI \times b)$

Recognize that the numerator is equal to beginning of period assets, that is:

$(TA_E - NI \times b) = TA_B$

Substituting this into the previous equation, we get:

Internal growth rate = $(NI \times b) / TA_B$

Which is equivalent to:

Internal growth rate = $(NI / TA_B) \times b$

Since $ROA_B = NI / TA_B$

The internal growth rate equation is:

Internal growth rate = $ROA_B \times b$

# *CHAPTER 5*
# INTRODUCTION TO VALUATION: THE TIME VALUE OF MONEY

**Answers to Concepts Review and Critical Thinking Questions**

**1.** The four parts are the present value (PV), the future value (FV), the discount rate (*r*), and the life of the investment (*t*).

**2.** Compounding refers to the growth of a dollar amount through time via reinvestment of interest earned. It is also the process of determining the future value of an investment. Discounting is the process of determining the value today of an amount to be received in the future.

**3.** Future values grow (assuming a positive rate of return); present values shrink.

**4.** The future value rises (assuming it's positive); the present value falls.

**5.** It would appear to be both deceptive and unethical to run such an ad without a disclaimer or explanation.

**6.** It's a reflection of the time value of money. TMCC gets to use the $24,099. If TMCC uses it wisely, it will be worth more than $100,000 in thirty years.

**7.** This will probably make the security less desirable. TMCC will only repurchase the security prior to maturity if it is to its advantage, i.e., interest rates decline. Given the drop in interest rates needed to make this viable for TMCC, it is unlikely the company will repurchase the security. This is an example of a "call" feature. Such features are discussed at length in a later chapter.

**8.** The key considerations would be: (1) Is the rate of return implicit in the offer attractive relative to other, similar risk investments? and (2) How risky is the investment; i.e., how certain are we that we will actually get the $100,000? Thus, our answer does depend on who is making the promise to repay.

**9.** The Treasury security would have a somewhat higher price because the Treasury is the strongest of all borrowers.

**10.** The price would be higher because, as time passes, the price of the security will tend to rise toward $100,000. This rise is just a reflection of the time value of money. As time passes, the time until receipt of the $100,000 grows shorter, and the present value rises. In 2019, the price will probably be higher for the same reason. We cannot be sure, however, because interest rates could be much higher, or TMCC's financial position could deteriorate. Either event would tend to depress the security's price.

**11.** To find the PV of a lump sum, we use:

$PV = FV / (1 + r)^t$
$PV = \$1{,}000{,}000 / (1.10)^{80} = \$488.19$

**12.** To find the FV of a lump sum, we use:

$FV = PV(1 + r)^t$
$FV = \$50(1.045)^{105} = \$5{,}083.71$

**13.** To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for $r$, we get:

$r = (FV / PV)^{1/t} - 1$
$r = (\$1{,}260{,}000 / \$150)^{1/112} - 1 = .0840$ or 8.40%

To find the FV of the first prize, we use:

$FV = PV(1 + r)^t$
$FV = \$1{,}260{,}000(1.0840)^{33} = \$18{,}056{,}409.94$

**14.** To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for $r$, we get:

$r = (FV / PV)^{1/t} - 1$
$r = (\$43{,}125 / \$1)^{1/113} - 1 = .0990$ or 9.90%

**15.** To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for $r$, we get:

$r = (FV / PV)^{1/t} - 1$
$r = (\$10{,}311{,}500 / \$12{,}377{,}500)^{1/4} - 1 = -4.46\%$

Notice that the interest rate is negative. This occurs when the FV is less than the PV.

*Intermediate*

**16.** To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for $r$, we get:

$r = (FV / PV)^{1/t} - 1$

*a.* $PV = \$100,000 / (1 + r)^{30} = \$24,099$
$r = (\$100,000 / \$24,099)^{1/30} - 1 = .0486$ or 4.86%

*b.* $PV = \$38,260 / (1 + r)^{12} = \$24,099$
$r = (\$38,260 / \$24,099)^{1/12} - 1 = .0393$ or 3.93%

*c.* $PV = \$100,000 / (1 + r)^{18} = \$38,260$
$r = (\$100,000 / \$38,260)^{1/18} - 1 = .0548$ or 5.48%

**17.** To find the PV of a lump sum, we use:

$PV = FV / (1 + r)^t$
$PV = \$170,000 / (1.12)^9 = \$61,303.70$

**18.** To find the FV of a lump sum, we use:

$FV = PV(1 + r)^t$

$FV = \$4,000(1.11)^{45} = \$43[illegible]$

$FV = \$4,000(1.11)^{35} = \$154,299.40$

Better start early!

**19.** We need to find the FV of a lump sum. However, the money will only be invested for six years, so the number of periods is six.

$FV = PV(1 + r)^t$
$FV = \$20,000(1.084)^6 = \$32,449.33$

**25.** In the previous problem, the cash flows are monthly and the compounding period is monthly. This assumption still holds. Since the cash flows are annual, we need to use the EAR to calculate the future value of annual cash flows. It is important to remember that you have to make sure the compounding periods of the interest rate is the same as the timing of the cash flows. In this case, we have annual cash flows, so we need the EAR since it is the true annual interest rate you will earn. So, finding the EAR:

$EAR = [1 + (APR / m)]^m - 1$
$EAR = [1 + (.10/12)]^{12} - 1 = .1047$ or 10.47%

Using the FVA equation, we get:

$FVA = C\{[(1 + r)^t - 1] / r\}$
$FVA = \$3,600[(1.1047^{30} - 1) / .1047] = \$647,623.45$

**26.** The cash flows are simply an annuity with four payments per year for four years, or 16 payments. We can use the PVA equation:

$PVA = C(\{1 - [1/(1 + r)]^t\} / r)$
$PVA = \$2,300\{[1 - (1/1.0065)^{16}] / .0065\} = \$34,843.71$

**27.** The cash flows are annual and the compounding period is quarterly, so we need to calculate the EAR to make the interest rate comparable with the timing of the cash flows. Using the equation for the EAR, we get:

$EAR = [1 + (APR / m)]^m - 1$
$EAR = [1 + (.11/4)]^4 - 1 = .1146$ or 11.46%

And now we use the EAR to find the PV of each cash flow as a lump sum and add them together:

$PV = \$725 / 1.1146 + \$980 / 1.1146^2 + \$1,360 / 1.1146^4 = \$2,320.36$



**28.** Here the cash flows are annual and the given interest rate is annual, so we can use the interest rate given. We simply find the PV of each cash flow and add them together.

$PV = \$1,650 / 1.0845 + \$4,200 / 1.0845^3 + \$2,430 / 1.0845^4 = \$6,570.86$

*Intermediate*

**29.** The total interest paid by First Simple Bank is the interest rate per period times the number of periods. In other words, the interest by First Simple Bank paid over 10 years will be:

$.07(10) = .7$

First Complex Bank pays compound interest, so the interest paid by this bank will be the FV factor of \$1, or:

$(1 + r)^{10}$

With the nonrefundable fee, the APR of the loan is simply the quoted APR since the fee is not considered part of the loan. So:

APR = 6.80%

EAR = $[1 + (.068/12)]^{12} - 1 = 7.02\%$

**65.** Be careful of interest rate quotations. The actual interest rate of a loan is determined by the cash flows. Here, we are told that the PV of the loan is $1,000, and the payments are $41.15 per month for three years, so the interest rate on the loan is:

PVA = $1,000 = $41.15$[\{1 - [1 / (1 + r)]^{36} \} / r ]$

Solving for $r$ with a spreadsheet, on a financial calculator, or by trial and error, gives:

$r$ = 2.30% per month

APR = 12(2.30%) = 27.61%

EAR = $(1 + .0230)^{12} - 1 = 31.39\%$

It's called add-on interest because the interest amount of the loan is added to the principal amount of the loan before the loan payments are calculated.

**66.** Here we are solving a two-step time value of money problem. Each question asks for a different possible cash flow to fund the same retirement plan. Each savings possibility has the same FV, that is, the PV of the retirement spending when your friend is ready to retire. The amount needed when your friend is ready to retire is:

PVA = $105,000{[1 – (1/1.07)^25] / .07} = $1,112,371.50

This amount is the same for all three parts of this question.

*a.* If your friend makes equal annual deposits into the account, this is an annuity with the FVA equal to the amount needed in retirement. The required savings each year will be:

FVA = $1,112,371.50 = $C[(1.07^{30} - 1) / .07]$
$C$ = $11,776.01

*b.* Here we need to find a lump sum savings amount. Using the FV for a lump sum equation, we get:

FV = $1,112,371.50 = PV$(1.07)^{30}$
PV = $146,129.04

| | N | I/Y | PV | PMT | FV |
|---|---|---|---|---|---|
| Enter | 30 × 12 | | $240,000 | ±$1,579.61 | |
| Solve for | | 0.5745% | | | |

APR = 0.5745% × 12 = 6.89%

| | NOM | EFF | C/Y |
|---|---|---|---|
| Enter | 6.89% | | 12 |
| Solve for | | 7.12% | |

Without refundable fee: APR = 6.80%

| | NOM | EFF | C/Y |
|---|---|---|---|
| Enter | 6.80% | | 12 |
| Solve for | | 7.02% | |

**65.**

| | N | I/Y | PV | PMT | FV |
|---|---|---|---|---|---|
| Enter | 36 | | $1,000 | ±$41.15 | |
| Solve for | | 2.30% | | | |

APR = 2.30% × 12 = 27.61%

| | NOM | EFF | C/Y |
|---|---|---|---|
| Enter | 27.61% | | 12 |
| Solve for | | 31.39% | |



**66.** What she needs at age 65

| | N | I/Y | PV | PMT | FV |
|---|---|---|---|---|---|
| Enter | 20 | 7% | | $105,000 | |
| Solve for | | | $1,112,371.50 | | |

*a.*

| | N | I/Y | PV | PMT | FV |
|---|---|---|---|---|---|
| Enter | 30 | 7% | | | $1,112,371.50 |
| Solve for | | | | $11,776.01 | |

*b.*

| | N | I/Y | PV | PMT | FV |
|---|---|---|---|---|---|
| Enter | 30 | 7% | | | $1,112,371.50 |
| Solve for | | | $146,129.04 | | |

*c.*

| | N | I/Y | PV | PMT | FV |
|---|---|---|---|---|---|
| Enter | 10 | 7% | $150,000 | | |
| Solve for | | | | | $295,072.70 |

At 65, she is short: $1,112,371.50 – 295,072.50 = $817,298.80

| | N | I/Y | PV | PMT | FV |
|---|---|---|---|---|---|
| Enter | 30 | 7% | | | ±$817,298.80 |
| Solve for | | | | $8,652.25 | |

Her employer will contribute $1,500 per year, so she must contribute:

$8,652.25 – 1,500 = $7,152.25 per year

**67.** Without fee:

| | N | I/Y | PV | PMT | FV |
|---|---|---|---|---|---|
| Enter | | 19.8% / 12 | $10,000 | ±$200 | |
| Solve for | 106.50 | | | | |

| | N | I/Y | PV | PMT | FV |
|---|---|---|---|---|---|
| Enter | | 6.8% / 12 | $10,000 | ±$200 | |
| Solve for | 57.99 | | | | |

With fee:

| | N | I/Y | PV | PMT | FV |
|---|---|---|---|---|---|
| Enter | | 6.8% / 12 | $10,200 | ±$200 | |
| Solve for | 59.35 | | | | |



**68.** Value at Year 4:

| | N | I/Y | PV | PMT | FV |
|---|---|---|---|---|---|
| Enter | 5 | 12% | $900 | | |
| Solve for | | | | | $1,586.11 |

| | N | I/Y | PV | PMT | FV |
|---|---|---|---|---|---|
| Enter | 4 | 12% | $900 | | |
| Solve for | | | | | $1,416.17 |

| | N | I/Y | PV | PMT | FV |
|---|---|---|---|---|---|
| Enter | 3 | 12% | $1,000 | | |
| Solve for | | | | | $1,404.93 |

| | N | I/Y | PV | PMT | FV |
|---|---|---|---|---|---|
| Enter | 2 | 12% | $1,000 | | |
| Solve for | | | | | $1,254.40 |

**10.** The term structure is based on pure discount bonds. The yield curve is based on coupon-bearing issues.

**11.** Bond ratings have a subjective factor to them. Split ratings reflect a difference of opinion among credit agencies.

**12.** As a general constitutional principle, the federal government cannot tax the states without their consent if doing so would interfere with state government functions. At one time, this principle was thought to provide for the tax-exempt status of municipal interest payments. However, modern court rulings make it clear that Congress can revoke the municipal exemption, so the only basis now appears to be historical precedent. The fact that the states and the federal government do not tax each other's securities is referred to as "reciprocal immunity."

**13.** Lack of transparency means that a buyer or seller can't see recent transactions, so it is much harder to determine what the best bid and ask prices are at any point in time.

**14.** Companies charge that bond rating agencies are pressuring them to pay for bond ratings. When a company pays for a rating, it has the opportunity to make its case for a particular rating. With an unsolicited rating, the company has no input.

**15.** A 100-year bond looks like a share of preferred stock. In particular, it is a loan with a life that almost certainly exceeds the life of the lender, assuming that the lender is an individual. With a junk bond, the credit risk can be so high that the borrower is almost certain to default, meaning that the creditors are very likely to end up as part owners of the business. In both cases, the "equity in disguise" has a significant tax advantage.



**Solutions to Questions and Problems**

*NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** The yield to maturity is the required rate of return on a bond expressed as a nominal annual interest rate. For noncallable bonds, the yield to maturity and required rate of return are interchangeable terms. Unlike YTM and required return, the coupon rate is not a return used as the interest rate in bond cash flow valuation, but is a fixed percentage of par over the life of the bond used to set the coupon payment amount. For the example given, the coupon rate on the bond is still 10 percent, and the YTM is 8 percent.

**2.** Price and yield move in opposite directions; if interest rates rise, the price of the bond will fall. This is because the fixed coupon payments determined by the fixed coupon rate are not as valuable when interest rates rise—hence, the price of the bond decreases.

**14.** This is a premium bond because it sells for more than 100% of face value. The current yield is:

Current yield = Annual coupon payment / Price = \$75/\$1,351.5625 = 5.978%

The YTM is located under the "Asked Yield" column, so the YTM is 4.47%.

The bid-ask spread is the difference between the bid price and the ask price, so:

Bid-Ask spread = 135:06 – 135:05 = 1/32

*Intermediate*

**15.** Here we are finding the YTM of semiannual coupon bonds for various maturity lengths. The bond price equation is:

$P = C(\text{PVIFA}_{R\%,t}) + \$1{,}000(\text{PVIF}_{R\%,t})$

X: $P_0 = \$80(\text{PVIFA}_{6\%,13}) + \$1{,}000(\text{PVIF}_{6\%,13}) = \$1{,}177.05$
$P_1 = \$80(\text{PVIFA}_{6\%,12}) + \$1{,}000(\text{PVIF}_{6\%,12}) = \$1{,}167.68$
$P_3 = \$80(\text{PVIFA}_{6\%,10}) + \$1{,}000(\text{PVIF}_{6\%,10}) = \$1{,}147.20$
$P_8 = \$80(\text{PVIFA}_{6\%,5}) + \$1{,}000(\text{PVIF}_{6\%,5}) = \$1{,}084.25$
$P_{12} = \$80(\text{PVIFA}_{6\%,1}) + \$1{,}000(\text{PVIF}_{6\%,1}) = \$1{,}018.87$
$P_{13} = \$1{,}000$

Y: $P_0 = \$60(\text{PVIFA}_{8\%,13}) + \$1{,}000(\text{PVIF}_{8\%,13}) = \$841.92$
$P_1 = \$60(\text{PVIFA}_{8\%,12}) + \$1{,}000(\text{PVIF}_{8\%,12}) = \$849.28$
$P_3 = \$60(\text{PVIFA}_{8\%,10}) + \$1{,}000(\text{PVIF}_{8\%,10}) = \$865.80$
$P_8 = \$60(\text{PVIFA}_{8\%,5}) + \$1{,}000(\text{PVIF}_{8\%,5}) = \$920.15$
$P_{12} = \$60(\text{PVIFA}_{8\%,1}) + \$1{,}000(\text{PVIF}_{8\%,1}) = \$981.48$
$P_{13} = \$1{,}000$



All else held equal, the premium over par value for a premium bond declines as maturity approaches, and the discount from par value for a discount bond declines as maturity approaches. This is called "pull to par." In both cases, the largest percentage price changes occur at the shortest maturity lengths.

Also, notice that the price of each bond when no time is left to maturity is the par value, even though the purchaser would receive the par value plus the coupon payment immediately. This is because we calculate the clean price of the bond.

$$t \ln\left(\frac{1+g}{1+R}\right) = \ln(0.5)$$

$$t = \frac{\ln(0.5)}{\ln\left(\frac{1 + g}{1 + R}\right)}$$

This expression will tell you the number of dividends that constitute one-half of the current stock price.

**27.** To find the value of the stock with two-stage dividend growth, consider that the present value of the first $t$ dividends is the present value of a growing annuity. Additionally, to find the price of the stock, we need to add the present value of the stock price at time $t$. So, the stock price today is:

$P_0$ = PV of $t$ dividends + PV($P_t$)

Using $g_1$ to represent the first growth rate and substituting the equation for the present value of a growing annuity, we get:

$$P_0 = D_1\left[\frac{1 - \left(\frac{1 + g_1}{1 + R}\right)^t}{R - g_1}\right] + PV(P_t)$$

Since the dividend in one year will increase at $g_1$, we can re-write the expression as:

$$P_0 = D_0(1 + g_1)\left[\frac{1 - \left(\frac{1 + g_1}{1 + R}\right)^t}{R - g_1}\right] + PV(P_t)$$



Now we can re-write the equation again as:

$$P_0 = \frac{D_0(1 + g_1)}{R - g_1}\left[1 - \left(\frac{1+g_1}{1+R}\right)^t\right] + PV(P_t)$$

To find the price of the stock at time $t$, we can use the constant dividend growth model, or:

$$P_t = \frac{D_{t+1}}{R - g_2}$$

The dividend at $t + 1$ will have grown at $g_1$ for $t$ periods, and at $g_2$ for one period, so:

$D_{t+1} = D_0(1 + g_1)^t(1 + g_2)$

So, we can re-write the equation as:

$$P_t = \frac{D(1 + g_1)^t(1 + g_2)}{R - g_2}$$

Next, we can find value today of the future stock price as:

$$PV(P_t) = \frac{D(1 + g_1)^t(1 + g_2)}{R - g_2} \times \frac{1}{(1 + R)^t}$$

which can be written as:

$$PV(P_t) = \left(\frac{1 + g_1}{1 + R}\right)^t \times \frac{D(1 + g_2)}{R - g_2}$$

Substituting this into the stock price equation, we get:

$$P_0 = \frac{D_0(1 + g_1)}{R - g_1}\left[1 - \left(\frac{1 + g_1}{1 + R}\right)^t\right] + \left(\frac{1 + g_1}{1 + R}\right)^t \times \frac{D(1 + g_2)}{R - g_2}$$

In this equation, the first term on the right hand side is the present value of the first $t$ dividends, and the second term is the present value of the stock price when constant dividend growth forever begins.



**28.** To find the expression when the growth rate for the first stage is exactly equal to the required return, consider we can find the present value of the dividends in the first stage as:

$$PV = \frac{D_0(1+g_1)^1}{(1+R)^1} + \frac{D_0(1+g_1)^2}{(1+R)^2} + \frac{D_0(1+g_1)^3}{(1+R)^3} + \ldots$$

Since $g_1$ is equal to $R$, each of the terns reduces to:

$PV = D_0 + D_0 + D_0 + \ldots.$
$PV = t \times D_0$

So, the expression for the price of a stock when the first growth rate is exactly equal to the required return is:

$$P_t = t \times D_0 + \frac{D_0 \times (1+g_1)^t \times (1+g_2)}{R - g_2}$$

# *CHAPTER 9*
# NET PRESENT VALUE AND OTHER INVESTMENT CRITERIA

**Answers to Concepts Review and Critical Thinking Questions**

**1.** A payback period less than the project's life means that the NPV is positive for a zero discount rate, but nothing more definitive can be said. For discount rates greater than zero, the payback period will still be less than the project's life, but the NPV may be positive, zero, or negative, depending on whether the discount rate is less than, equal to, or greater than the IRR. The discounted payback includes the effect of the relevant discount rate. If a project's discounted payback period is less than the project's life, it must be the case that NPV is positive.

**2.** If a project has a positive NPV for a certain discount rate, then it will also have a positive NPV for a zero discount rate; thus, the payback period must be less than the project life. Since discounted payback is calculated at the same discount rate as is NPV, if NPV is positive, the discounted payback period must be less than the project's life. If NPV is positive, then the present value of future cash inflows is greater than the initial investment cost; thus PI must be greater than 1. If NPV is positive for a certain discount rate R, then it will be zero for some larger discount rate R*; thus the IRR must be greater than the required return.



**3.** *a.* Payback period is simply the accounting break-even point of a series of cash flows. To actually compute the payback period, it is assumed that any cash flow occurring during a given period is realized continuously throughout the period, and not at a single point in time. The payback is then the point in time for the series of cash flows when the initial cash outlays are fully recovered. Given some predetermined cutoff for the payback period, the decision rule is to accept projects that payback before this cutoff, and reject projects that take longer to payback.

*b.* The worst problem associated with payback period is that it ignores the time value of money. In addition, the selection of a hurdle point for payback period is an arbitrary exercise that lacks any steadfast rule or method. The payback period is biased towards short-term projects; it fully ignores any cash flows that occur after the cutoff point.

*c.* Despite its shortcomings, payback is often used because (1) the analysis is straightforward and simple and (2) accounting numbers and estimates are readily available. Materiality considerations often warrant a payback analysis as sufficient; maintenance projects are another example where the detailed analysis of other methods is often not needed. Since payback is biased towards liquidity, it may be a useful and appropriate analysis method for short-term projects where cash management is most important.

**4.** *a.* The discounted payback is calculated the same as is regular payback, with the exception that each cash flow in the series is first converted to its present value. Thus discounted payback provides a measure of financial/economic break-even because of this discounting, just as regular payback provides a measure of accounting break-even because it does not discount the cash flows. Given some predetermined cutoff for the discounted payback period, the decision rule is to accept projects whose discounted cash flows payback before this cutoff period, and to reject all other projects.

*b.* The equation for the IRR of the project is:

$0 = –\$45{,}000{,}000 + \$78{,}000{,}000/(1+IRR) – \$14{,}000{,}000/(1+IRR)^2$

From Descartes rule of signs, we know there are potentially two IRRs since the cash flows change signs twice. From trial and error, the two IRRs are:

IRR = 53.00%, –79.67%

When there are multiple IRRs, the IRR decision rule is ambiguous. Both IRRs are correct, that is, both interest rates make the NPV of the project equal to zero. If we are evaluating whether or not to accept this project, we would not want to use the IRR to make our decision.

**15.** The profitability index is defined as the PV of the cash inflows divided by the PV of the cash outflows. The equation for the profitability index at a required return of 10 percent is:

$PI = [\$7{,}300/1.1 + \$6{,}900/1.1^2 + \$5{,}700/1.1^3] / \$14{,}000 = 1.187$

The equation for the profitability index at a required return of 15 percent is:

$PI = [\$7{,}300/1.15 + \$6{,}900/1.15^2 + \$5{,}700/1.15^3] / \$14{,}000 = 1.094$

The equation for the profitability index at a required return of 22 percent is:

$PI = [\$7{,}300/1.22 + \$6{,}900/1.22^2 + \$5{,}700/1.22^3] / \$14{,}000 = 0.983$

We would accept the project if the required return were 10 percent or 15 percent since the PI is greater than one. We would reject the project if the required return were 22 percent since the PI is less than one.



**16.** *a.* The profitability index is the PV of the future cash flows divided by the initial investment. The cash flows for both projects are an annuity, so:

$PI_I = \$27{,}000(PVIFA_{10\%,3}) / \$53{,}000 = 1.267$

$PI_{II} = \$9{,}100(PVIFA_{10\%,3}) / \$16{,}000 = 1.414$

The profitability index decision rule implies that we accept project II, since $PI_{II}$ is greater than the $PI_I$.

*b.* The NPV of each project is:

$NPV_I = –\$53{,}000 + \$27{,}000(PVIFA_{10\%,3}) = \$14{,}145.00$

$NPV_{II} = –\$16{,}000 + \$9{,}100(PVIFA_{10\%,3}) = \$6{,}630.35$

The NPV decision rule implies accepting Project I, since the $NPV_I$ is greater than the $NPV_{II}$.

**27.** The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the project is:

$0 = \$20{,}000 - \$26{,}000 / (1 + IRR) + \$13{,}000 / (1 + IRR)^2$

Even though it appears there are two IRRs, a spreadsheet, financial calculator, or trial and error will not give an answer. The reason is that there is no real IRR for this set of cash flows. If you examine the IRR equation, what we are really doing is solving for the roots of the equation. Going back to high school algebra, in this problem we are solving a quadratic equation. In case you don't remember, the quadratic equation is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this case, the equation is:

$$x = \frac{-(-26{,}000) \pm \sqrt{(-26{,}000)^2 - 4(20{,}000)(13{,}000)}}{2(26{,}000)}$$

The square root term works out to be:

$676{,}000{,}000 - 1{,}040{,}000{,}000 = -364{,}000{,}000$

The square root of a negative number is a complex number, so there is no real number solution, meaning the project has no real IRR.



**28.** First, we need to find the future value of the cash flows for the one year in which they are blocked by the government. So, reinvesting each cash inflow for one year, we find:

Year 2 cash flow = $\$205{,}000(1.04) = \$213{,}200$
Year 3 cash flow = $\$265{,}000(1.04) = \$275{,}600$
Year 4 cash flow = $\$346{,}000(1.04) = \$359{,}840$
Year 5 cash flow = $\$220{,}000(1.04) = \$228{,}800$

So, the NPV of the project is:

$NPV = -\$450{,}000 + \$213{,}200/1.11^2 + \$275{,}600/1.11^3 + \$359{,}840/1.11^4 + \$228{,}800/1.11^5$
$NPV = -\$2{,}626.33$

And the IRR of the project is:

$0 = -\$450{,}000 + \$213{,}200/(1 + IRR)^2 + \$275{,}600/(1 + IRR)^3 + \$359{,}840/(1 + IRR)^4 + \$228{,}800/(1 + IRR)^5$

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 10.89%

While this may look like a MIRR calculation, it is not an MIRR, rather it is a standard IRR calculation. Since the cash inflows are blocked by the government, they are not available to the company for a period of one year. Thus, all we are doing is calculating the IRR based on when the cash flows actually occur for the company.

**Calculator Solutions**

**7.**

| | |
|---|---|
| **CFo** | –$34,000 |
| **C01** | $16,000 |
| **F01** | 1 |
| **C02** | $18,000 |
| **F02** | 1 |
| **C03** | $15,000 |
| **F03** | 1 |

IRR CPT
20.97%

**8.**

| | |
|---|---|
| **CFo** | –$34,000 |
| **C01** | $16,000 |
| **F01** | 1 |
| **C02** | $18,000 |
| **F02** | 1 |
| **C03** | $15,000 |
| **F03** | 1 |

I = 11%
NPV CPT
$5,991.49

| | |
|---|---|
| **CFo** | –$34,000 |
| **C01** | $16,000 |
| **F01** | 1 |
| **C02** | $18,000 |
| **F02** | 1 |
| **C03** | $15,000 |
| **F03** | 1 |

I = 30%
NPV CPT
–$4,213.93



**9.**

| | |
|---|---|
| **CFo** | –$138,000 |
| **C01** | $28,500 |
| **F01** | 9 |

I = 8%
NPV CPT
$40,036.31

| | |
|---|---|
| **CFo** | –$138,000 |
| **C01** | $28,500 |
| **F01** | 9 |

I = 20%
NPV CPT
–$23,117.45

| | |
|---|---|
| **CFo** | –$138,000 |
| **C01** | $28,500 |
| **F01** | 9 |

IRR CPT
14.59%

*Project B*

| | |
|---|---|
| **CF**o | –$43,000 |
| **C01** | $7,000 |
| **F01** | 1 |
| **C02** | $13,800 |
| **F02** | 1 |
| **C03** | $24,000 |
| **F03** | 1 |
| **C04** | $26,000 |
| **F04** | 1 |

IRR CPT
18.84%

| | |
|---|---|
| **CF**o | –$43,000 |
| **C01** | $7,000 |
| **F01** | 1 |
| **C02** | $13,800 |
| **F02** | 1 |
| **C03** | $24,000 |
| **F03** | 1 |
| **C04** | $26,000 |
| **F04** | 1 |

I = 11%
NPV CPT
$9,182.29

*Crossover rate*

| | |
|---|---|
| **CF**o | $0 |
| **C01** | $16,000 |
| **F01** | 1 |
| **C02** | $4,100 |
| **F02** | 1 |
| **C03** | –$11,600 |
| **F03** | 1 |
| **C04** | –$16,600 |
| **F04** | 1 |

IRR CPT
15.30%



**13.** *Project X*

| | |
|---|---|
| **CF**o | –$15,000 |
| **C01** | $8,150 |
| **F01** | 1 |
| **C02** | $5,050 |
| **F02** | 1 |
| **C03** | $6,800 |
| **F03** | 1 |

I = 0%
NPV CPT
$5,000.00

| | |
|---|---|
| **CF**o | –$15,000 |
| **C01** | $8,150 |
| **F01** | 1 |
| **C02** | $5,050 |
| **F02** | 1 |
| **C03** | $6,800 |
| **F03** | 1 |

I = 15%
NPV CPT
$376.59

| | |
|---|---|
| **CF**o | –$15,000 |
| **C01** | $8,150 |
| **F01** | 1 |
| **C02** | $5,050 |
| **F02** | 1 |
| **C03** | $6,800 |
| **F03** | 1 |

I = 25%
NPV CPT
–$1,766.40

**15.**

| CFo | $0 |
|---|---|
| C01 | $7,300 |
| F01 | 1 |
| C02 | $6,900 |
| F02 | 1 |
| C03 | $5,700 |
| F03 | 1 |

I = 10%
NPV CPT
$16,621.34

| CFo | $0 |
|---|---|
| C01 | $7,300 |
| F01 | 1 |
| C02 | $6,900 |
| F02 | 1 |
| C03 | $5,700 |
| F03 | 1 |

I = 15%
NPV CPT
$15,313.06

| CFo | $0 |
|---|---|
| C01 | $7,300 |
| F01 | 1 |
| C02 | $6,900 |
| F02 | 1 |
| C03 | $5,700 |
| F03 | 1 |

I = 22%
NPV CPT
$13,758.49

@10%: PI = $16,621.34 / $14,000 = 1.187
@15%: PI = $15,313.06 / $14,000 = 1.094
@22%: PI = $13,758.49 / $14,000 = 0.983

**16.** *Project I*

| CFo | $0 |
|---|---|
| C01 | $27,000 |
| F01 | 3 |

I = 10%
NPV CPT
$67,145.00

| CFo | –$53,000 |
|---|---|
| C01 | $27,000 |
| F01 | 3 |

I = 10%
NPV CPT
$14,145.00

PI = $67,145.00 / $53,000 = 1.267

*Project II*

| CFo | $0 |
|---|---|
| C01 | $9,100 |
| F01 | 3 |

I = 10%
NPV CPT
$22,630.35

| CFo | –$16,000 |
|---|---|
| C01 | $9,100 |
| F01 | 3 |

I = 10%
NPV CPT
$6,630.35

PI = $22,630.35 / $16,000 = 1.414

**17.**

*CF(A)*

*c.*

| Cfo | –$300,000 |
|---|---|
| C01 | $20,000 |
| F01 | 1 |
| C02 | $50,000 |
| F02 | 2 |
| C03 | $390,000 |
| F03 | 1 |

I = 15%
NPV CPT
$11,058.07

*d.*

| CFo | –$300,000 |
|---|---|
| C01 | $20,000 |
| F01 | 1 |
| C02 | $50,000 |
| F02 | 2 |
| C03 | $390,000 |
| F03 | 1 |

IRR CPT
16.20%

*e.*

| CFo | $0 |
|---|---|
| C01 | $20,000 |
| F01 | 1 |
| C02 | $50,000 |
| F02 | 2 |
| C03 | $390,000 |
| F03 | 1 |

I = 15%
NPV CPT
$311,058.07

PI = $311,058.07 / $300,000 = 1.037

**13.** First we will calculate the annual depreciation of the new equipment. It will be:

Annual depreciation = $560,000/5
Annual depreciation = $112,000

Now, we calculate the aftertax salvage value. The aftertax salvage value is the market price minus (or plus) the taxes on the sale of the equipment, so:

Aftertax salvage value = $MV + (BV - MV)t_c$

Very often the book value of the equipment is zero as it is in this case. If the book value is zero, the equation for the aftertax salvage value becomes:

Aftertax salvage value = $MV + (0 - MV)t_c$
Aftertax salvage value = $MV(1 - t_c)$

We will use this equation to find the aftertax salvage value since we know the book value is zero. So, the aftertax salvage value is:

Aftertax salvage value = $85,000(1 – 0.34)
Aftertax salvage value = $56,100

Using the tax shield approach, we find the OCF for the project is:

OCF = $165,000(1 – 0.34) + 0.34($112,000)
OCF = $146,980

Now we can find the project NPV. Notice we include the NWC in the initial cash outlay. The recovery of the NWC occurs in Year 5, along with the aftertax salvage value.

NPV = $-\$560,000 - 29,000 + \$146,980(PVIFA_{10\%,5}) + [(\$56,100 + 29,000) / 1.10^5]$
NPV = $21,010.24



**14.** First we will calculate the annual depreciation of the new equipment. It will be:

Annual depreciation charge = $720,000/5
Annual depreciation charge = $144,000

The aftertax salvage value of the equipment is:

Aftertax salvage value = $75,000(1 – 0.35)
Aftertax salvage value = $48,750

Using the tax shield approach, the OCF is:

OCF = $260,000(1 – 0.35) + 0.35($144,000)
OCF = $219,400

**16.** To calculate the EAC of the project, we first need the NPV of the project. Notice that we include the NWC expenditure at the beginning of the project, and recover the NWC at the end of the project. The NPV of the project is:

NPV = –$270,000 – 25,000 – $42,000(PVIFA$_{11\%,5}$) + $25,000/1.11$^5$ = –$435,391.39

Now we can find the EAC of the project. The EAC is:

EAC = –$435,391.39 / (PVIFA$_{11\%,5}$) = –$117,803.98

**17.** We will need the aftertax salvage value of the equipment to compute the EAC. Even though the equipment for each product has a different initial cost, both have the same salvage value. The aftertax salvage value for both is:

Both cases: aftertax salvage value = $40,000(1 – 0.35) = $26,000

To calculate the EAC, we first need the OCF and NPV of each option. The OCF and NPV for Techron I is:

OCF = –$67,000(1 – 0.35) + 0.35($290,000/3) = –9,716.67

NPV = –$290,000 – $9,716.67(PVIFA$_{10\%,3}$) + ($26,000/1.10$^3$) = –$294,629.73

EAC = –$294,629.73 / (PVIFA$_{10\%,3}$) = –$118,474.97

And the OCF and NPV for Techron II:

OCF = –$35,000(1 – 0.35) + 0.35($510,000/5) = $12,950

NPV = –$510,000 + $12,950(PVIFA$_{10\%,5}$) + ($26,000/1.10$^5$) = –$444,765.36

EAC = –$444,765.36 / (PVIFA$_{10\%,5}$) = –$117,327.98

The two milling machines have unequal lives, so they can only be compared by expressing both on an equivalent annual basis, which is what the EAC method does. Thus, you prefer the Techron II because it has the lower (less negative) annual cost.

**18.** To find the bid price, we need to calculate all other cash flows for the project, and then solve for the bid price. The aftertax salvage value of the equipment is:

Aftertax salvage value = $70,000(1 – 0.35) = $45,500

Now we can solve for the necessary OCF that will give the project a zero NPV. The equation for the NPV of the project is:

NPV = 0 = –$940,000 – 75,000 + OCF(PVIFA$_{12\%,5}$) + [($75,000 + 45,500) / 1.12$^5$]

**25.** A kilowatt hour is 1,000 watts for 1 hour. A 60-watt bulb burning for 500 hours per year uses 30,000 watt hours, or 30 kilowatt hours. Since the cost of a kilowatt hour is $0.101, the cost per year is:

Cost per year = 30($0.101)
Cost per year = $3.03

The 60-watt bulb will last for 1,000 hours, which is 2 years of use at 500 hours per year. So, the NPV of the 60-watt bulb is:

NPV = –$0.50 – $3.03($PVIFA_{10\%,2}$)
NPV = –$5.76

And the EAC is:

EAC = –$5.83 / ($PVIFA_{10\%,2}$)
EAC = –$3.32

Now we can find the EAC for the 15-watt CFL. A 15-watt bulb burning for 500 hours per year uses 7,500 watts, or 7.5 kilowatts. And, since the cost of a kilowatt hour is $0.101, the cost per year is:

Cost per year = 7.5($0.101)
Cost per year = $0.7575

The 15-watt CFL will last for 12,000 hours, which is 24 years of use at 500 hours per year. So, the NPV of the CFL is:

NPV = –$3.50 – $0.7575($PVIFA_{10\%,24}$)
NPV = –$10.31

And the EAC is:

EAC = –$10.85 / ($PVIFA_{10\%,24}$)
EAC = –$1.15

Thus, the CFL is much cheaper. But see our next two questions.



**26.** To solve the EAC algebraically for each bulb, we can set up the variables as follows:

W = light bulb wattage
C = cost per kilowatt hour
H = hours burned per year
P = price the light bulb

The number of watts use by the bulb per hour is:

WPH = W / 1,000

And the kilowatt hours used per year is:

KPY = WPH × H

**Solutions to Questions and Problems**

*NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** *a.* The total variable cost per unit is the sum of the two variable costs, so:

Total variable costs per unit = $5.43 + 3.13
Total variable costs per unit = $8.56

*b.* The total costs include all variable costs and fixed costs. We need to make sure we are including all variable costs for the number of units produced, so:

Total costs = Variable costs + Fixed costs
Total costs = $8.56(280,000) + $720,000
Total costs = $3,116,800

*c.* The cash breakeven, that is the point where cash flow is zero, is:

$Q_C$ = $720,000 / ($19.99 – 8.56)
$Q_C$ = 62,992.13 units

And the accounting breakeven is:

$Q_A$ = ($720,000 + 220,000) / ($19.99 – 8.56)
$Q_A$ = 82,239.72 units



**2.** The total costs include all variable costs and fixed costs. We need to make sure we are including all variable costs for the number of units produced, so:

Total costs = ($24.86 + 14.08)(120,000) + $1,550,000
Total costs = $6,222,800

The marginal cost, or cost of producing one more unit, is the total variable cost per unit, so:

Marginal cost = $24.86 + 14.08
Marginal cost = $38.94

The average cost per unit is the total cost of production, divided by the quantity produced, so:

Average cost = Total cost / Total quantity
Average cost = $6,222,800/120,000
Average cost = $51.86

Minimum acceptable total revenue = 5,000($38.94)
Minimum acceptable total revenue = $194,700

Additional units should be produced only if the cost of producing those units can be recovered.

**3.** The base-case, best-case, and worst-case values are shown below. Remember that in the best-case, sales and price increase, while costs decrease. In the worst-case, sales and price decrease, and costs increase.

| Scenario | Unit Sales | Unit Price | Unit Variable Cost | Fixed Costs |
|---|---|---|---|---|
| Base | 95,000 | $1,900.00 | $240.00 | $4,800,000 |
| Best | 109,250 | $2,185.00 | $204.00 | $4,080,000 |
| Worst | 80,750 | $1,615.00 | $276.00 | $5,520,000 |

**4.** An estimate for the impact of changes in price on the profitability of the project can be found from the sensitivity of NPV with respect to price: $\Delta NPV/\Delta P$. This measure can be calculated by finding the NPV at any two different price levels and forming the ratio of the changes in these parameters. Whenever a sensitivity analysis is performed, all other variables are held constant at their base-case values.



**5.** *a.* To calculate the accounting breakeven, we first need to find the depreciation for each year. The depreciation is:

Depreciation = $724,000/8
Depreciation = $90,500 per year

And the accounting breakeven is:

$Q_A$ = ($780,000 + 90,500)/($43 – 29)
$Q_A$ = 62,179 units

To calculate the accounting breakeven, we must realize at this point (and only this point), the OCF is equal to depreciation. So, the DOL at the accounting breakeven is:

DOL = 1 + FC/OCF = 1 + FC/D
DOL = 1 + [$780,000/$90,500]
DOL = 9.919

*b.* We will use the tax shield approach to calculate the OCF. The OCF is:

$OCF_{base} = [(P – v)Q – FC](1 – t_c) + t_cD$
$OCF_{base} = [(\$43 – 29)(90,000) – \$780,000](0.65) + 0.35(\$90,500)$
$OCF_{base} = \$343,675$

Now that we know the product price, we can use the accounting breakeven equation to find the depreciation. Doing so, we find the annual depreciation must be:

$Q_A = (FC + D)/(P - v)$
$15,500 = (\$140,000 + D)/(\$34.61 - 24)$
Depreciation = $24,394

We now know the annual depreciation amount. Assuming straight-line depreciation is used, the initial investment in equipment must be five times the annual depreciation, or:

Initial investment = 5($24,394) = $121,970

The PV of the OCF must be equal to this value at the financial breakeven since the NPV is zero, so:

$\$121,970 = OCF(PVIFA_{16\%,5})$
OCF = $37,250.69

We can now use this OCF in the financial breakeven equation to find the financial breakeven sales quantity is:

$Q_F = (\$140,000 + 37,250.69)/(\$34.61 - 24)$
$Q_F = 16,712$

**11.** We know that the DOL is the percentage change in OCF divided by the percentage change in quantity sold. Since we have the original and new quantity sold, we can use the DOL equation to find the percentage change in OCF. Doing so, we find:

DOL = %ΔOCF / %ΔQ

Solving for the percentage change in OCF, we get:

%ΔOCF = (DOL)(%ΔQ)
%ΔOCF = 3.40[(70,000 – 65,000)/65,000]
%ΔOCF = .2615 or 26.15%

The new level of operating leverage is lower since FC/OCF is smaller.

**12.** Using the DOL equation, we find:

DOL = 1 + FC / OCF
3.40 = 1 + $130,000/OCF
OCF = $54,167

The percentage change in quantity sold at 58,000 units is:

%ΔQ = (58,000 – 65,000) / 65,000
%ΔQ = –.1077 or –10.77%

*c.* The definition of the financial breakeven is where the NPV of the project is zero. If this is true, then the IRR of the project is equal to the required return. It is impossible to state the payback period, except to say that the payback period must be less than the length of the project. Since the discounted cash flows are equal to the initial investment, the undiscounted cash flows are greater than the initial investment, so the payback must be less than the project life.

**17.** Using the tax shield approach, the OCF at 110,000 units will be:

$OCF = [(P - v)Q - FC](1 - t_C) + t_C(D)$
$OCF = [(\$32 - 19)(110{,}000) - 210{,}000](0.66) + 0.34(\$490{,}000/4)$
$OCF = \$846{,}850$

We will calculate the OCF at 111,000 units. The choice of the second level of quantity sold is arbitrary and irrelevant. No matter what level of units sold we choose, we will still get the same sensitivity. So, the OCF at this level of sales is:

$OCF = [(\$32 - 19)(111{,}000) - 210{,}000](0.66) + 0.34(\$490{,}000/4)$
$OCF = \$855{,}430$

The sensitivity of the OCF to changes in the quantity sold is:

$Sensitivity = \Delta OCF/\Delta Q = (\$846{,}850 - 855{,}430)/(110{,}000 - 111{,}000)$
$\Delta OCF/\Delta Q = +\$8.58$

OCF will increase by $5.28 for every additional unit sold.

**18.** At 110,000 units, the DOL is:

$DOL = 1 + FC/OCF$
$DOL = 1 + (\$210{,}000/\$846{,}850)$
$DOL = 1.2480$

The accounting breakeven is:

$Q_A = (FC + D)/(P - v)$
$Q_A = [\$210{,}000 + (\$490{,}000/4)]/(\$32 - 19)$
$Q_A = 25{,}576$

And, at the accounting breakeven level, the DOL is:

$DOL = 1 + [\$210{,}000/(\$490{,}000/4)]$
$DOL = 2.7143$

**19.** *a.* The base-case, best-case, and worst-case values are shown below. Remember that in the best-case, sales and price increase, while costs decrease. In the worst-case, sales and price decrease, and costs increase.

| Scenario | Unit sales | Variable cost | Fixed costs |
|---|---|---|---|
| Base | 190 | $11,200 | $410,000 |
| Best | 209 | $10,080 | $369,000 |
| Worst | 171 | $12,320 | $451,000 |

Using the tax shield approach, the OCF and NPV for the base case estimate is:

$OCF_{base} = [(\$18,000 - 11,200)(190) - \$410,000](0.65) + 0.35(\$1,700,000/4)$
$OCF_{base} = \$722,050$

$NPV_{base} = -\$1,700,000 + \$722,050(PVIFA_{12\%,4})$
$NPV_{base} = \$493,118.10$

The OCF and NPV for the worst case estimate are:

$OCF_{worst} = [(\$18,000 - 12,320)(171) - \$451,000](0.65) + 0.35(\$1,700,000/4)$
$OCF_{worst} = \$486,932$

$NPV_{worst} = -\$1,700,000 + \$486,932(PVIFA_{12\%,4})$
$NPV_{worst} = -\$221,017.41$

And the OCF and NPV for the best case estimate are:

$OCF_{best} = [(\$18,000 - 10,080)(209) - \$369,000](0.65) + 0.35(\$1,700,000/4)$
$OCF_{best} = \$984,832$

$NPV_{best} = -\$1,700,000 + \$984,832(PVIFA_{12\%,4})$
$NPV_{best} = \$1,291,278.83$

*b.* To calculate the sensitivity of the NPV to changes in fixed costs we choose another level of fixed costs. We will use fixed costs of $420,000. The OCF using this level of fixed costs and the other base case values with the tax shield approach, we get:

$OCF = [(\$18,000 - 11,200)(190) - \$410,000](0.65) + 0.35(\$1,700,000/4)$
$OCF = \$715,550$

And the NPV is:

$NPV = -\$1,700,000 + \$715,550(PVIFA_{12\%,4})$
$NPV = \$473,375.32$

The sensitivity of NPV to changes in fixed costs is:

$\Delta NPV/\Delta FC = (\$493,118.10 - 473,375.32)/(\$410,000 - 420,000)$
$\Delta NPV/\Delta FC = -\$1.974$

For every dollar FC increase, NPV falls by $1.974.

*c.* The cash breakeven is:

$Q_C = FC/(P - v)$
$Q_C = \$410,000/(\$18,000 - 11,200)$
$Q_C = 60$

*d.* The accounting breakeven is:

$Q_A = (FC + D)/(P - v)$
$Q_A = [\$410,000 + (\$1,700,000/4)]/(\$18,000 - 11,200)$
$Q_A = 123$

At the accounting breakeven, the DOL is:

DOL = 1 + FC/OCF
DOL = 1 + ($410,000/$425,000) = 1.9647

For each 1% increase in unit sales, OCF will increase by 1.9647%.

**20.** The marketing study and the research and development are both sunk costs and should be ignored. We will calculate the sales and variable costs first. Since we will lose sales of the expensive clubs and gain sales of the cheap clubs, these must be accounted for as erosion. The total sales for the new project will be:

| Sales | | |
|---|---|---|
| New clubs | $750 × 51,000 = | $38,250,000 |
| Exp. clubs | $1,200 × (–11,000) = | –13,200,000 |
| Cheap clubs | $420 × 9,500 = | 3,990,000 |
| | | $29,040,000 |



For the variable costs, we must include the units gained or lost from the existing clubs. Note that the variable costs of the expensive clubs are an inflow. If we are not producing the sets anymore, we will save these variable costs, which is an inflow. So:

| Var. costs | | |
|---|---|---|
| New clubs | –$330 × 51,000 = | –$16,830,000 |
| Exp. clubs | –$650 × (–11,000) = | 7,150,000 |
| Cheap clubs | –$190 × 9,500 = | –1,805,000 |
| | | –$11,485,000 |

The pro forma income statement will be:

| | |
|---|---|
| Sales | $29,040,000 |
| Variable costs | 11,485,000 |
| Costs | 8,100,000 |
| Depreciation | 3,200,000 |
| EBT | $6,255,000 |
| Taxes | 2,502,000 |
| Net income | $3,753,000 |

*d.* The implicit assumption in the previous analysis is that each car depreciates by the same dollar amount.

**24.** *a.* The cash flow per plane is the initial cost divided by the breakeven number of planes, or:

Cash flow per plane = $13,000,000,000 / 249
Cash flow per plane = $52,208,835

*b.* In this case the cash flows are a perpetuity. Since we know the cash flow per plane, we need to determine the annual cash flow necessary to deliver a 20 percent return. Using the perpetuity equation, we find:

PV = C /R
$13,000,000,000 = C / .20
C = $2,600,000,000

This is the total cash flow, so the number of planes that must be sold is the total cash flow divided by the cash flow per plane, or:

Number of planes = $2,600,000,000 / $52,208,835
Number of planes = 49.80 or about 50 planes per year

*c.* In this case the cash flows are an annuity. Since we know the cash flow per plane, we need to determine the annual cash flow necessary to deliver a 20 percent return. Using the present value of an annuity equation, we find:

$PV = C(PVIFA_{20\%,10})$
$\$13,000,000,000 = C(PVIFA_{20\%,10})$
$C = \$3,100,795,839$

This is the total cash flow, so the number of planes that must be sold is the total cash flow divided by the cash flow per plane, or:

Number of planes = $3,100,795,839 / $52,208,835
Number of planes = 59.39 or about 60 planes per year



*Challenge*

**25.** *a.* The tax shield definition of OCF is:

$OCF = [(P - v)Q - FC](1 - t_C) + t_C D$

Rearranging and solving for Q, we find:

$(OCF - t_C D)/(1 - t_C) = (P - v)Q - FC$
$Q = \{FC + [(OCF - t_C D)/(1 - t_C)]\}/(P - v)$

The sensitivity of changes in the OCF to quantity sold is:

$\Delta OCF/\Delta Q = (\$968,600 – 940,700)/(36,000 – 35,000)$
$\Delta OCF/\Delta Q = +\$27.90$

The NPV at this level of sales is:

$NPV = –\$3,200,000 – \$360,000 + \$968,600(PVIFA_{13\%,5}) + [\$360,000 + \$500,000(1 – .38)]/1.13^5$
$NPV = \$210,439.36$

And the sensitivity of NPV to changes in the quantity sold is:

$\Delta NPV/\Delta Q = (\$210,439.36 – 112,308.60))/(36,000 – 35,000)$
$\Delta NPV/\Delta Q = +\$98.13$

You wouldn't want the quantity to fall below the point where the NPV is zero. We know the NPV changes $98.13 for every unit sale, so we can divide the NPV for 35,000 units by the sensitivity to get a change in quantity. Doing so, we get:

$\$112,308.60 = \$98.13(\Delta Q)$
$\Delta Q = 1,144$

For a zero NPV, we need to decrease sales by 1,144 units, so the minimum quantity is:

$Q_{Min} = 35,000 – 1,144$
$Q_{Min} = 33,856$



**29.** At the cash breakeven, the OCF is zero. Setting the tax shield equation equal to zero and solving for the quantity, we get:

$OCF = 0 = [(\$230 – 185)Q_C – \$450,000](0.62) + 0.38(\$3,200,000/5)$
$Q_C = 1,283$

The accounting breakeven is:

$Q_A = [\$450,000 + (\$3,200,000/5)]/(\$230 – 185)$
$Q_A = 24,222$

From Problem 28, we know the financial breakeven is 33,856 units.

**30.** Using the tax shield approach to calculate the OCF, the DOL is:

DOL = 1 + [$450,000(1 – 0.38) – 0.38($3,200,000/5)]/ $940,700
DOL = 1.03806

Thus a 1% rise leads to a 1.03806% rise in OCF. If Q rises to 36,000, then

The percentage change in quantity is:

ΔQ = (36,000 – 35,000)/35,000 = .02857 or 2.857%

So, the percentage change in OCF is:

%ΔOCF = 2.857%(1.03806)
%ΔOCF = 2.9659%

From Problem 26:
ΔOCF/OCF = ($968,600 – 940,700)/$940,700
ΔOCF/OCF = 0.029659

In general, if Q rises by 1,000 units, OCF rises by 2.9659%.

*b.* Using the equation to calculate variance, we find:

Variance = 1/4[(.07 – .116)$^2$ + (–.12 – .116)$^2$ + (.11 – .116)$^2$ + (.38 – .116)$^2$ + (.14 – .116)$^2$]
Variance = 0.032030

So, the standard deviation is:

Standard deviation = (0.03230)$^{1/2}$ = 0.1790 or 17.90%

**10.** *a.* To calculate the average real return, we can use the average return of the asset, and the average inflation in the Fisher equation. Doing so, we find:

$(1 + R) = (1 + r)(1 + h)$

$\bar{r} = (1.160/1.035) - 1 = .0783$ or 7.83%

*b.* The average risk premium is simply the average return of the asset, minus the average risk-free rate, so, the average risk premium for this asset would be:

$\overline{RP} = \bar{R} - \bar{R}_f = .1160 - .042 = .0740$ or 7.40%

**11.** We can find the average real risk-free rate using the Fisher equation. The average real risk-free rate was:

$(1 + R) = (1 + r)(1 + h)$

$\bar{r}_f = (1.042/1.035) - 1 = .0068$ or 0.68%

And to calculate the average real risk premium, we can subtract the average risk-free rate from the average real return. So, the average real risk premium was:

$\overline{rp} = \bar{r} - \bar{r}_f = 7.83\% - 0.68\% = 7.15\%$

**12.** T-bill rates were highest in the early eighties. This was during a period of high inflation and is consistent with the Fisher effect.

**22.** To find the real return we need to use the Fisher equation. Re-writing the Fisher equation to solve for the real return, we get:

$r = [(1 + R)/(1 + h)] - 1$

So, the real return each year was:

| Year | T-bill return | Inflation | Real return |
|---|---|---|---|
| 1973 | 0.0729 | 0.0871 | –0.0131 |
| 1974 | 0.0799 | 0.1234 | –0.0387 |
| 1975 | 0.0587 | 0.0694 | –0.0100 |
| 1976 | 0.0507 | 0.0486 | 0.0020 |
| 1977 | 0.0545 | 0.0670 | –0.0117 |
| 1978 | 0.0764 | 0.0902 | –0.0127 |
| 1979 | 0.1056 | 0.1329 | –0.0241 |
| 1980 | 0.1210 | 0.1252 | –0.0037 |
| | 0.6197 | 0.7438 | –0.1120 |

*a.* The average return for T-bills over this period was:

Average return = 0.619 / 8
Average return = .0775 or 7.75%

And the average inflation rate was:

Average inflation = 0.7438 / 8
Average inflation = .0930 or 9.30%



*b.* Using the equation for variance, we find the variance for T-bills over this period was:

$$\text{Variance} = 1/7[(.0729 - .0775)^2 + (.0799 - .0775)^2 + (.0587 - .0775)^2 + (.0507 - .0775)^2 + (.0545 - .0775)^2 + (.0764 - .0775)^2 + (.1056 - .0775)^2 + (.1210 - .0775)^2]$$

Variance = 0.000616

And the standard deviation for T-bills was:

Standard deviation = $(0.000616)^{1/2}$
Standard deviation = 0.0248 or 2.48%

The variance of inflation over this period was:

$$\text{Variance} = 1/7[(.0871 - .0930)^2 + (.1234 - .0930)^2 + (.0694 - .0930)^2 + (.0486 - .0930)^2 + (.0670 - .0930)^2 + (.0902 - .0930)^2 + (.1329 - .0930)^2 + (.1252 - .0930)^2]$$

Variance = 0.000971

And the standard deviation of inflation was:

Standard deviation = $(0.000971)^{1/2}$
Standard deviation = 0.0312 or 3.12%

And the probability that T-bill returns will be less than 0 percent is:

$z_4 = (0\% - 3.8)/3.1\% = -1.2258$

$\Pr(R \leq 0) \approx 11.01\%$

*c*. The probability that the return on long-term corporate bonds will be less than –4.18 percent is:

$z_5 = (-4.18\% - 6.2)/8.4\% = -1.2357$

$\Pr(R \leq -4.18\%) \approx 10.83\%$

And the probability that T-bill returns will be greater than 10.56 percent is:

$z_6 = (10.56\% - 3.8)/3.1\% = 2.1806$

$\Pr(R \geq 10.56\%) = 1 - \Pr(R \leq 10.56\%) = 1 - .9823 \approx 1.46\%$

We also know the total portfolio weight must be one, so the weight of the risk-free asset must be one minus the asset weight we know, or:

$1 = w_A + w_B + w_C + w_{Rf} = 1 - .210 - .320 - .324074 - w_{Rf}$

$w_{Rf} = .145926$

So, the dollar investment in the risk-free asset must be:

Invest in risk-free asset = .145926($1,000,000) = $145,925.93

*Challenge*

**25.** We are given the expected return of the assets in the portfolio. We also know the sum of the weights of each asset must be equal to one. Using this relationship, we can express the expected return of the portfolio as:

$E(R_p) = .185 = w_X(.172) + w_Y(.136)$
$.185 = w_X(.172) + (1 - w_X)(.136)$
$.185 = .172w_X + .136 - .136w_X$
$.049 = .036w_X$
$w_X = 1.36111$

And the weight of Stock Y is:

$w_Y = 1 - 1.36111$
$w_Y = -.36111$

The amount to invest in Stock Y is:

Investment in Stock Y = –.36111($100,000)
Investment in Stock Y = –$36,111.11

A negative portfolio weight means that you short sell the stock. If you are not familiar with short selling, it means you borrow a stock today and sell it. You must then purchase the stock at a later date to repay the borrowed stock. If you short sell a stock, you make a profit if the stock decreases in value.

To find the beta of the portfolio, we can multiply the portfolio weight of each asset times its beta and sum. So, the beta of the portfolio is:

$\beta_P = 1.36111(1.40) + (-.36111)(0.95)$
$\beta_P = 1.56$

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