<u>Illustration</u> <u>1</u>: To show how a Cartesian coordinate system is transformed into a polar coordinate system

$$x = g(u,v) \qquad \text{Given,}$$
$$y = h(u,v) \qquad \iint f(x,y)\, dA = \iint f(u,v)\, dA$$

Suppose we need to determine *dA* where *dA* would represent the area of a small parallelogram in the *U* and *V* co-ordinate system

$$\vec{r} = \langle x, y \rangle$$
$$\vec{r} = \langle g(u,v), h(u,v) \rangle$$

So, the area of a small parallelogram would be determined by

$$\frac{\partial \vec{r}}{\partial u} \times \Delta u \quad \text{and} \quad \frac{\partial \vec{r}}{\partial v} \times \Delta v$$

iii) $J' = \dfrac{\partial(r,\phi)}{\partial(x,y)} = \dfrac{1}{r}$

$= \dfrac{\partial(x,y)}{\partial(r,\phi)} = r$

Accoringin to given

$J \times J' = 1$

$\therefore \cancel{r} \times \dfrac{1}{\cancel{r}} = 1$

$\therefore$ Proved.

PROPERTY 2: CHAIN RULE

$$\frac{\partial(u,v)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(r,s)} \times \frac{\partial(r,s)}{\partial(x,y)}$$

R.H.S.:=

$$\begin{vmatrix} \frac{\partial u}{\partial r} & \frac{\partial u}{\partial s} \\ \frac{\partial v}{\partial r} & \frac{\partial v}{ds} \end{vmatrix} \times \begin{vmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} \\ \frac{\partial s}{\partial x} & \frac{\partial s}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial u}{\partial r} \times \frac{\partial r}{\partial x} + \frac{\partial u}{\partial s} \times \frac{\partial s}{\partial x} & \frac{\partial u}{\partial r} \times \frac{\partial r}{\partial y} + \frac{\partial u}{\partial s} \times \frac{\partial s}{\partial y} \\ \frac{\partial v}{\partial r} \times \frac{\partial r}{\partial x} + \frac{\partial v}{\partial s} \times \frac{\partial s}{\partial x} & \frac{\partial v}{\partial r} \times \frac{\partial r}{\partial y} + \frac{\partial v}{\partial s} \times \frac{\partial s}{\partial y} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \frac{\partial(u,v)}{\partial(x,y)} = L.H.S.$$

Hence Proved.