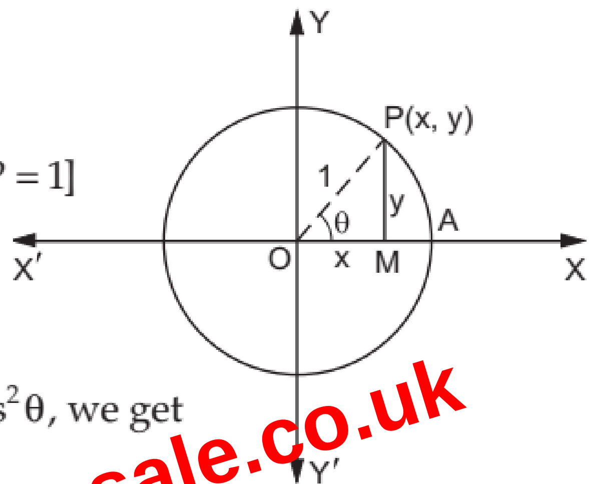
PROOF Let $X'OX$ and YOY' be the coordinate axes. Taking O as the centre and a unit radius, draw a circle, meeting OX at A.

Let $P(x, y)$ be a point on the circle with $\angle AOP = \theta$. Join OP.

Draw $PM \perp OA$.

Then, $\cos\theta = x$ and $\sin\theta = y$.

(i) From right ΔOMP, we have



$$OM^2 + PM^2 = OP^2$$

$$\Rightarrow \quad x^2 + y^2 = 1$$

$[\because OM = x,\ PM = y \text{ and } OP = 1]$

$$\Rightarrow \quad \cos^2\theta + \sin^2\theta = 1. \quad \ldots \text{(i)}$$

$[\because x = \cos\theta,\ y = \sin\theta]$

This proves (i).

(ii) Dividing both sides of (i) by $\cos^2\theta$, we get

$$1 + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\Rightarrow \quad 1 + \tan^2\theta = \sec^2\theta \quad \left[\because \frac{\sin\theta}{\cos\theta} = \tan\theta \text{ and } \frac{1}{\cos\theta} = \sec\theta\right].$$

(iii) Dividing both sides of (i) by $\sin^2\theta$, we get

$$\frac{\cos^2\theta}{\sin^2\theta} + 1 = \frac{1}{\sin^2\theta}$$

$$\Rightarrow \quad \cot^2\theta + 1 = \operatorname{cosec}^2\theta \quad \left[\because \frac{\cos\theta}{\sin\theta} = \cot\theta \text{ and } \frac{1}{\sin\theta} = \operatorname{cosec}\theta\right]$$

NEGATIVE ARC LENGTH *If a point moves in a circle then the arc length covered by it is said to be positive or negative depending on whether the point moves in the anticlockwise or clockwise direction respectively.*

(iii) $180° = \pi^c$

$$\Rightarrow \quad 600° = \left(\frac{\pi}{180} \times 600\right)^c = \left(\frac{10\pi}{3}\right)^c.$$

$$\therefore \quad \cot(-600°) = -\cot 600° \qquad [\because \cot(-\theta) = -\cot\theta]$$

$$= -\cot\left(\frac{10\pi}{3}\right)$$

$$= -\cot\left(3\pi + \frac{\pi}{3}\right)$$

$$= -\cot\frac{\pi}{3} \qquad [\because \cot(n\pi + \theta) = \cot\theta]$$

$$= -\frac{1}{\sqrt{3}}.$$

EXAMPLE 5 *Find the value of*

(i) $\cos 15\pi$ *(ii)* $\sin 16\pi$ *(iii)* $\cos(-\pi)$

(iv) $\sin 5\pi$ *(v)* $\tan\left(\frac{5\pi}{4}\right)$ *(vi)* $\sec 6\pi$

SOLUTION (i) $\cos 15\pi = \cos(14\pi + \pi)$

$$= \cos\pi \qquad [\because \cos(2n\pi + \theta) = \cos\theta]$$

$$= -1.$$

(ii) $\sin 16\pi = \sin(16\pi + 0)$

$$= \sin 0° \qquad [\because \sin(2n\pi + \theta) = \sin\theta]$$

$$= 0.$$

(iii) $\cos(-\pi) = \cos\pi \qquad [\because \cos(-\theta) = \cos\theta]$

$$= -1.$$

(iv) $\sin 5\pi = \sin(4\pi + \pi)$

$$= \sin\pi \qquad [\because \sin(2n\pi + \theta) = \sin\theta]$$

$$= 0.$$

(v) $\tan\frac{5\pi}{4} = \tan\left(\pi + \frac{\pi}{4}\right)$

$$= \tan\frac{\pi}{4} \qquad [\because \tan(n\pi + \theta) = \tan\theta]$$

$$= 1.$$