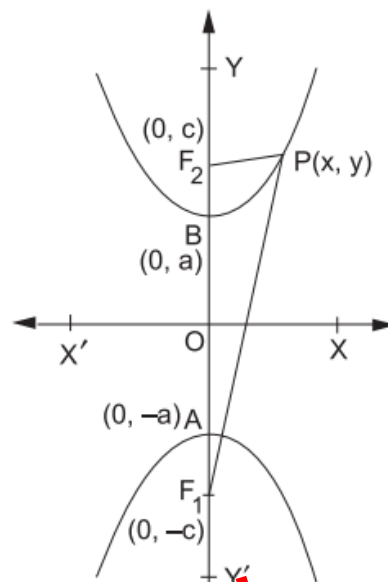


SUMMARY OF THE RESULTS ON VERTICAL HYPERBOLA

- (i) The standard equation of a vertical hyperbola is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.
- (ii) Its centre is $O(0, 0)$.
- (iii) YOY' is the *transverse axis* and $X'OX$ is the *conjugate axis*.
- (iv) Its foci are $F_1(0, -c)$ and $F_2(0, c)$, i.e., $F_1(0, -ae)$ and $F_2(0, ae)$.
- (v) Its vertices are $A(0, -a)$ and $B(0, a)$.
- (vi) Its eccentricity is, $e = \frac{c}{a} = \frac{\sqrt{a^2 + b^2}}{a}$.
- (vii) Length of the transverse axis = $2a$ and its equation is $x = 0$.
- (viii) Length of the conjugate axis = $2b$ and its equation is $y = 0$.
- (ix) Length of its latus rectum = $\frac{b^2}{a}$.



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Page 7 of 15

SOLVED EXAMPLES

EXAMPLE 1 Find the lengths of the axes; the coordinates of the vertices and the foci; the eccentricity and length of the latus rectum of the hyperbola

$$\frac{x^2}{36} - \frac{y^2}{64} = 1.$$

SOLUTION The equation of the given hyperbola is $\frac{x^2}{36} - \frac{y^2}{64} = 1$.

Comparing the given equation with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we get

$$a^2 = 36 \text{ and } b^2 = 64.$$

$$\therefore a = 6, b = 8 \text{ and } c = \sqrt{a^2 + b^2} = \sqrt{36 + 64} = \sqrt{100} = 10.$$

- (i) Length of the transverse axis = $2a = (2 \times 6)$ units = 12 units.
Length of the conjugate axis = $2b = (2 \times 8)$ units = 16 units.
- (ii) The coordinates of the vertices are $A(-a, 0)$ and $B(a, 0)$, i.e., $A(-6, 0)$ and $B(6, 0)$.

Conventional Practice Questions

Find the (i) lengths of the axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity and (v) length of the latus rectum of each of the following the hyperbola:

1. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

2. $\frac{x^2}{25} - \frac{y^2}{4} = 1$

3. $x^2 - y^2 = 1$

4. $3x^2 - 2y^2 = 6$

5. $25x^2 - 9y^2 = 225$

6. $24x^2 - 25y^2 = 600$

7. $\frac{y^2}{16} - \frac{x^2}{49} = 1$

8. $\frac{y^2}{9} - \frac{x^2}{27} = 1$

9. $3y^2 - x^2 = 108$

10. $5y^2 - 9x^2 = 36$

11. Find the equation of the hyperbola with vertices at $(\pm 6, 0)$ and foci at $(\pm 8, 0)$.
12. Find the equation of the hyperbola with vertices at $(0, \pm 5)$ and foci at $(0, \pm 8)$.
13. Find the equation of the hyperbola whose foci are $(\pm\sqrt{29}, 0)$ and the transverse axis is of the length 10.
14. Find the equation of the hyperbola whose foci are $(\pm 5, 0)$ and the conjugate axis is of the length 8. Also, find its eccentricity.
15. Find the equation of the hyperbola whose foci are $(\pm 3\sqrt{5}, 0)$ and the length of the latus rectum is 8 units.
16. Find the equation of the hyperbola whose vertices are $(\pm 2, 0)$ and the eccentricity is 2.
17. Find the equation of the hyperbola whose foci are $(\pm\sqrt{5}, 0)$ and the eccentricity is $\sqrt{\frac{5}{3}}$.
18. Find the equation of the hyperbola, the length of whose latus rectum is 4 and the eccentricity is 3.
19. Find the equation of the hyperbola with eccentricity $\sqrt{2}$ and the distance between whose foci is 16.