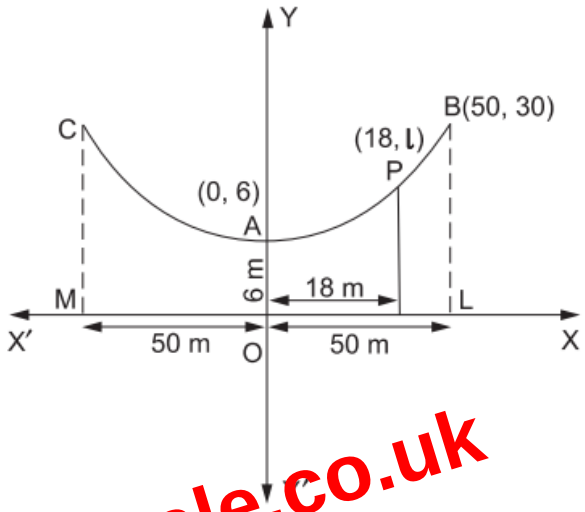
EXAMPLE 7 *The cable of a uniformly loaded suspension bridge hangs in the form of a parabola. The roadway which is horizontal and* 100 m *long is supported by vertical wires attached to the cable, the longest wire being* 30 m *and the shortest being* 6 m. *Find the length of a supporting wire attached to the roadway* 18 m *from the middle.*

SOLUTION Let $X'OX$ be the x-axis, taken along the roadway, and let YOY' be the y-axis.

Let CAB be the bridge in the form of a parabola having its vertex at $A(0, 6)$ such that A is the centre of the bridge.

Let BL and CM be the longest vertical wires, each of length 30 m, and let OA be the shortest vertical wire, 6 m in length.



The equation of the parabolic form of the bridge is

$$x^2 = 4a(y - 6) \qquad \ldots \text{(i)}$$

It passes through $B(50, 30)$.

Putting $x = 50$ and $y = 30$ in (i), we get

$$2500 = 4a(30 - 6) \quad \Rightarrow \quad a = \frac{2500}{96}.$$

Putting $a = \dfrac{2500}{96}$ in (i), we get

$$x^2 = 4 \times \frac{2500}{96}(y - 6) \quad \Rightarrow \quad 6x^2 = 625y - 3750. \qquad \ldots \text{(ii)}$$

Let l m be the length of the supporting wire 18 m from the middle.

Then, $P(18, l)$ must satisfy (ii).

$$\therefore \quad 6 \times 18 \times 18 = 625l - 3750 \quad \Rightarrow \quad 625l = 5694$$
$$\Rightarrow \quad l = 9.11.$$

Hence, the length of the supporting wire 18 m from the middle is 9.11 m.