$\therefore \quad AB + BC = AC.$

This shows that the given points are collinear.

EXAMPLE 4 *Find the equation of the curve formed by the set of all points whose distances from the points* $(3, 4, -5)$ *and* $(-2, 1, 4)$ *are equal.*

SOLUTION Let $P(x, y, z)$ be any point on the given curve, and let $A(3, 4, -5)$ and $B(-2, 1, 4)$ be the given points.

Then, $PA = PB$

$\Rightarrow \quad PA^2 = PB^2$

$\Rightarrow \quad (x-3)^2 + (y-4)^2 + (z+5)^2 = (x+2)^2 + (y-1)^2 + (z-4)^2$

$\Rightarrow \quad 10x + 6y - 18z - 29 = 0.$

Hence, the required curve is $10x + 6y - 18z - 29 = 0$.

EXAMPLE 5 *Find the equation of the curve formed by the set of all those points the sum of whose distances from the points* $A(4, 0, 0)$ *and* $B(-4, 0, 0)$ *is* 10 *units.*

SOLUTION Let $P(x, y, z)$ be an arbitrary point on the given curve. Then,

$PA + PB = 10$

$\Rightarrow \quad \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$

$\Rightarrow \quad \sqrt{(x+4)^2 + y^2 + z^2} = 10 - \sqrt{(x-4)^2 + y^2 + z^2} \quad \ldots \text{(i)}$

$\Rightarrow \quad (x+4)^2 + y^2 + z^2 = 100 + (x-4)^2 + y^2 + z^2 - 20\sqrt{(x-4)^2 + y^2 + z^2}$

[on squaring both sides of (i)]

$\Rightarrow \quad 16x = 100 - 20\sqrt{(x-4)^2 + y^2 + z^2}$

$\Rightarrow \quad 5\sqrt{(x-4)^2 + y^2 + z^2} = (25 - 4x)$

$\Rightarrow \quad 25[(x-4)^2 + y^2 + z^2] = 625 + 16x^2 - 200x$

$\Rightarrow \quad 9x^2 + 25y^2 + 25z^2 - 225 = 0.$

Hence, the required equation of the curve is

$$9x^2 + 25y^2 + 25z^2 - 225 = 0.$$

Answers

1. $(-1, 2, 1)$ **2.** $(3, -2, -1), (4, -5, 1)$ **3.** $(8, -14, 9)$

4. $1 : 2$ **5.** $3 : 2$ (externally) **6.** $3 : 2, (2, 0, 1)$

7. $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$ **8.** $2 : 3, \left(\frac{13}{5}, \frac{-9}{5}, \frac{-2}{5}\right)$ **9.** $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$

10. $(1, -8, 9)$ **11.** $(-2, 5, 8)$ **12.** $a = -2, b = -8, c = 2$

13. $(1, 2, 3); (3, 4, 5); (-1, 6, -7)$

HINTS TO SOME SELECTED QUESTIONS

2. Let P and Q be the points of trisection of AB. Then, P divides AB in the ratio $1 : 2$ and Q divides AB in the ratio $2 : 1$.

7. Suppose the xy-plane divides AB in the ratio $\lambda : 1$.

Then, $\frac{6\lambda + 1}{\lambda + 1} = 0 \Rightarrow \lambda = -\frac{1}{6}$.

Divide AB in the ratio $1 : (-6)$.

8. D divides BC in the ratio $AB : AC$.

❑