Exercises solved with Gauss sum

Exercice 1. Calculate these sums:

$$a)\ S_1 = 1 + 2 + 3 + ... + 100; \tag{1}$$

$$b)\ S_2 = 2 + 4 + 6 + ... + 100; \tag{2}$$

$$c)\ S_3 = 1 + 3 + 5 + ... + 99; \tag{3}$$

Solutions:

a) $S_1 = 100 \cdot (100 + 1) : 2 = 100 \cdot 101 : 2 = 10100 : 2 = 5050;$

b) Firstly, we will compute the numbers of terms: (100-2):2+1=98:2+1=50 after this we will find the value of sum;
$S_2 = (100 + 2) \cdot 50 : 2 = 102 \cdot 50 : 2 = 100 \cdot 51 : 2 = 5100 : 2 = 2550;$

c) The number of terms is:(99 - 1):2+1= 98:2+1=50;
In these conditions, we can find the sum: $S_3 = (1 + 99) \cdot 50 : 2 = 100 \cdot 50 : 2 = 5000 : 2 = 2500;$

Exercise 2. Compute these sums:

$$a)\ S_4 = 79 + 82 + 85 + ... + 184; \tag{4}$$

$$b)\ S_5 = 7 + 16 + 25 + ... + 862; \tag{5}$$

$$c)\ S_6 = 100 + 107 + 114 + ... + 436; \tag{6}$$

Solutions:

a) We will find firstly the numbers of terms: (184-79):3+1= 105:3+1= 36. Now, we can compute the sum:
$S_4 = (79 + 184) \cdot 36 : 2 = 263 \cdot 18 = 4734;$

b) The number of terms is: (862-7):9+1=855:9+1=95+1=96. In these conditions we will compute the sum:
$S_5 = (7 + 862) \cdot 96 : 2 = 869 \cdot 48 = 41712;$

c) The sum have 436-100+1=336+1=337 terms, and we can compute it in this way:
$S_6 = (100 + 436) \cdot 337 : 2 = 536 \cdot 337 : 2 = 536 : 2 \cdot 337 = 268 \cdot 337 = 90316;$

Exercise 3. Find these sums:

$$a)\ S_7 = 99 + 105 + 111 + ... + 363; \tag{7}$$

$$b)\ S_8 = 81 + 86 + 91 + ... + 361; \tag{8}$$

$$c)\ S_9 = 2 + 9 + 16 + ... + 583; \tag{9}$$

Solutions:

a) Firstly, we will find the numbers of terms : (363-99):6+1= 264:6+1= 44+1= 45. After this we can compute the sum:
$S_7 = (99 + 363) \cdot 45 : 2 = 462 \cdot 45 : 2 = 462 : 2 \cdot 45 = 231 \cdot 45 = 10395;$

b) The sum have (361-81):5+1=280:5+1=56+1=57 terms. Because we know the numbers of terms, we can compute the sum:
$S_8 = (81 + 361) \cdot 57 : 2 = 442 \cdot 57 : 2 = 442 : 2 \cdot 57 = 221 \cdot 57 = 12597;$

c) Firstly, we will compute the numbers of terms: (583-2):7+1= 581:7+1= 83+1= 84, after this we can find the sum:
$S_9 = (2 + 593) \cdot 84 : 2 = 595 \cdot 84 : 2 = 595 \cdot 42 = 24990;$