

→ Let $X \neq \emptyset$ defined by $d: X \times X \rightarrow \mathbb{R}$
 $d(x, y) = 0 \quad \forall x, y \in X$ is called trivial
metric or indiscrete metric.

Pseudo metric: X is a non empty set,
 d is a function, $d: X \times X \rightarrow \mathbb{R}$ is said
to be pseudometric, if

$$(i) \quad d(x, y) \geq 0 \quad \forall x, y \in X$$

$$(ii) \quad d(x, x) = 0 \quad \forall x \in X$$

$$(iii) \quad d(x, y) = d(y, x) \quad \forall x, y \in X$$

$$(iv) \quad d(x, z) \leq d(x, y) + d(y, z) \quad \forall x, y, z \in X$$

Remark: Every metric is a pseudometric
but not conversely.

→ Let $X = \mathbb{R}$, $d: \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$d(x, y) = |x^2 - y^2|$$

we can show that it is a pseudometric
but it is not a metric.

$$\text{If } d(x, y) = 0 \Rightarrow |x^2 - y^2| = 0$$

$$\Rightarrow x^2 - y^2 = 0 \Rightarrow x^2 = y^2 \Rightarrow x = \pm y$$

i.e., if $x = -y$, then also $d(x, y) = 0$

$\Rightarrow d$ is not a metric