

(Mathematics)

(M.A/M.Sc Part-I) Algebra

Roll No: _ Time Allowed : 2:25 hrs Max. Marks :

9

8

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9

Attempt FOUR Questions in all. Select TWO Questions from Section A and TWO Questions from Section B. All Questions carry equal marks.

SUBJECTIVE PART

SECTION-A

- 3- a) Let G be a group, H a subgroup and K a normal subgroup of G then show that $H/_{H\cap K} \cong HK/_{K}$ b) Prove that a group of prime order is abelian. Is a group of prime order cyclic group? Verify your answer. 4,1,3
- a) Define a Permutation and a Transposition. Write all subgroups of S_3 . Specify which one is normal and which one is not and why? 2,3,3
 - b) Let H and K be subgroups of a group G. Prove that the product HK is a subgroup of G if and only if HK = KH.
- 5- a) If G is a group of order n divisible by a prime p. Prove that the number of sylow p-subgroups of G is esale.co.uk 1 + tp, where $t \in N \cup \{0\}$ and 1 + tp divides n. 9
 - b) Prove that Centre of a finite p-group is non-trivial.
- 6- a) Show that ring \mathbb{Z} of integers is a proviple

Then prove that $R_{\text{kerd}} \cong \text{Im} \Phi$. b) Let R, R' be ring 8

- 7- a) Suppose T : V \rightarrow W is a linear transformation from a finite dimensional vector space V into a vector space W. Show that dim V = Rank of T + Nullity of T. b) If V and W are of dimension m and n respectively over F. Then prove that Hom (V, W) is of
 - dimension 'mn' over F.
- 8- a) Define Similar Matrices. Prove that any two similary matrices have same eigen values.

 $A = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ b) Check the matrix

for diagonalizability. If yes, find a matrix Q such that $Q^{-1}AQ$ is a diagonal matrix.

M.A/M.Sc-I(11/A) (MTH-II) (N)