

Log...

Differentiate the following function w.r.t 'x'

$$\textcircled{1} \left(1 + \frac{1}{x}\right)^x$$

$$\text{Let } y = \left(1 + \frac{1}{x}\right)^x$$

$$\log y = \log \left(1 + \frac{1}{x}\right)^x$$

$$\log y = x \cdot \log \left(1 + \frac{1}{x}\right)$$

$$\frac{1}{y} \frac{dy}{dx} = x \frac{d}{dx} \left[\log \left(1 + \frac{1}{x}\right) \right] + \log \left(1 + \frac{1}{x}\right) \frac{d}{dx} (x)$$

$$= x \cdot \frac{1}{\left(1 + \frac{1}{x}\right)} \frac{d}{dx} \left(1 + \frac{1}{x}\right) + \log \left(1 + \frac{1}{x}\right) \cdot 1$$

$$= x \cdot \frac{1}{\left(\frac{x+1}{x}\right)} \left[\frac{d}{dx} (1) + \frac{d}{dx} \left(\frac{1}{x}\right) \right] + \log \left(1 + \frac{1}{x}\right)$$

$$= x \times \frac{x}{(x+1)} \left[0 + (-1)x^{-2} \right] + \log \left(1 + \frac{1}{x}\right)$$

$$= \frac{x^2}{(x+1)} \left[-\frac{1}{x^2} \right] + \log \left(1 + \frac{1}{x}\right)$$

$$\frac{1}{y} \frac{dy}{dx} = -\frac{1}{x+1} + \log \left(1 + \frac{1}{x}\right)$$

$$\frac{dy}{dx} = y \left[-\frac{1}{x+1} + \log \left(1 + \frac{1}{x}\right) \right]$$

$$\boxed{\frac{dy}{dx} = \left(1 + \frac{1}{x}\right)^x \left[-\frac{1}{x+1} + \log \left(1 + \frac{1}{x}\right) \right]}$$