(4) $\tan^{-1}\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

Let $y = \tan^{-1}\sqrt{\dfrac{1-\cos x}{1+\cos x}}$ — (1)

$$\frac{1-\cos x}{1+\cos x} = \frac{2\sin^2 x/2}{2\cos^2 x/2}$$

$$\frac{1-\cos x}{1+\cos x} = \tan^2 x/2$$

$$\therefore y = \tan^{-1}(\tan^2 x/2)$$

$$\therefore y = x/2 \quad \text{— (1)}$$

Differentiate Equation (1) with respect to 'x'

$$\therefore \frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(x)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2} \times 1$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{1}{2}}$$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\theta = x$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= \cos^2 x - (1-\cos^2 x) \\ &= \cos^2 x - 1 + \cos^2 x \\ &= 2\cos^2 x - 1\end{aligned}$$

$\cos 2x + 1 = 2\cos^2 x$

$\cos x + 1 = 2\cos^2 x/2$

$1+\cos x = 2\cos^2 x/2$

similarly

$1-\cos x = 2\sin^2 x/2$

⑤ $\cos^{-1}\left(\log \frac{x}{2}\right)$

Solution: let $y = \cos^{-1}\left(\log \frac{x}{2}\right)$ —①

Differentiate equation ① with respect to x

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-\left(\log \frac{x}{2}\right)^2}} \frac{d}{dx}\left[\log \frac{x}{2}\right]$$

$$\therefore \frac{dy}{dx} = -\frac{1}{\sqrt{1-\frac{(\log x)^2}{(\log 2)^2}}} \frac{d}{dx}\left[\frac{\log x}{\log 2}\right]$$ --- [By change of base law]

$$\frac{dy}{dx} = -\frac{1}{\sqrt{\frac{(\log 2)^2-(\log x)^2}{(\log 2)^2}}} \times \frac{1}{\log 2}\frac{d}{dx}(\log x)$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{\frac{(\log 2)^2-(\log x)^2}{(\log 2)^2}}} \times \frac{1}{\log 2} \times \frac{1}{x}.$$

$$\frac{dy}{dx} = -\frac{\cancel{\log 2}}{\sqrt{(\log 2)^2-(\log x)^2}} \times \frac{1}{\cancel{\log 2}} \times \frac{1}{x}$$

$$\frac{dy}{dx} = -\frac{1}{x\sqrt{(\log 2)^2-(\log x)^2}}$$