13. If $x^3 + ax^2 + bx + 6$ has $x - 2$ as a factor and leaves a remainder 3 when divided by $x - 3$ find a and b

Solution:

$$\text{Let } p(x) = x^3 + ax^2 + bx + 6$$

Since $(x - 2)$ is a factor

$$p(2) = 0$$

$$\therefore (2)^3 + a(2)^2 + b(2) + 6 = 0$$

$$4a + 2b = -14$$

$$2a + b = -7 \qquad \text{-----(1)}$$

Since the remainder is 3 when divided by $(x - 3)$

$$p(3) = 3$$

$$(3)^3 + a(3)^2 + b(3) + 6 = 3$$

$$9a + 3b = -30$$

$$3a + b = -10 \qquad \text{----(2)}$$

(1) $2a + b = -7$

(2) $3a + b = -10$

$$-a = 3 \quad \text{or} \quad a = -3$$

Substitute a = -3 in equation (2)

$$3a + b = -10$$

$$3(-3) + b = -10$$

$$-9 + b = -10$$

$$b = -10 + 9 \quad b = -1$$

$\therefore a = -3, \quad b = -1$

14. Show by factor theorem $x - y$, $y - z$ and $z - x$ are the factors of