

11. $\frac{x^2 + x + 1}{(x-2)^2 (x+2)}$

Solution: Let $\frac{x^2 + x + 1}{(x-2)^2 (x+2)} = \frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

$$3x^2 - 1 = A(x-2) + B(x-2) + C$$

Put $x = 2$, $12 - 1 = C \Rightarrow C = 11$

Coefficient of $x^2 \Rightarrow 3 = A$

Put $x = 0$, $-1 = 4A - 2B + C$

$$-1 = 12 - 2B + 11$$

$$-2B = -24$$

$$B = 12$$

$\therefore$ The partial fractions are $\frac{3}{(x-2)} + \frac{13}{(x-2)^2} + \frac{11}{(x-2)^3}$

12. $\frac{x^2}{(x+3)(x^2+1)}$

Solution: Let $\frac{x^2}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}$

$$x^2 = A(x^2+1) + (Bx+C)(x+3)$$

Put $x = -3$, $9 = A(10) \Rightarrow A = \frac{9}{10}$

Put $x = 0$, $0 = A + 3C \Rightarrow C = \frac{-3}{10}$

Equating the coefficient of x^2

$$1 = A + B \Rightarrow B = \frac{1}{10}$$

$\therefore$ The partial fractions are $\frac{9}{10(x+3)} + \frac{\frac{1}{10}x - \frac{3}{10}}{x^2+1}$

That is, $\frac{9}{10(x+3)} + \frac{1}{10} \left(\frac{x-3}{x^2+1} \right)$