3. **Find the values of a and b in each of the following such that the polynomials P (x) and Q (x) have the given G.C.D (H.C.F)**

	P (x)	Q (x)	G.C.D
(i)	$(x^2 + 3x + 2)$ $(x^2 + x + a)$	$(x^2 - 3x + 2)$ $(x^2 - 3x + b)$	$(x + 1)$ $(x - 2)$
(ii)	$(x^2 + 3x + 2)$ $(x^2 - 4x + a)$	$(x^2 - 6x + 9)$ $(x^2 + 4x + b)$	$(x + 2)$ $(x - 3)$

Solution:

(i)
$$P(x) = (x^2 + 3x + 2)(x^2 + x + a)$$
$$= (x + 1)(x + 2)(x^2 + x + a)$$
$$Q(x) = (x^2 - 3x + 2)(x^2 - 3x + b)$$
$$= (x - 1)(x - 2)(x^2 - 3x + b)$$
$$\textbf{G.C.D} = \textbf{(}x + 1\textbf{)(}x - 2\textbf{)}$$

$\therefore$ $x^2 + x + a$ is divided exactly by $(x - 2)$

$$(2)^2 + 2 + a = 0$$
$$\boldsymbol{a} = \textbf{– 6}$$

Similarly $(x + 1)$ divides $x^2 - 3x + b$ exactly

$$(-1)^2 - 3(-1) + b = 0$$
$$\textbf{b} = \textbf{4}$$

(ii)
$$P(x) = (x^2 + 3x + 2)(x^2 - 4x + a)$$
$$= (x + 1)(x + 2)(x^2 - 4x + a)$$
$$Q(x) = (x^2 - 6x + 9)(x^2 + 4x + b)$$
$$= (x - 3)(x - 3)(x^2 + 4x + b)$$
$$\text{G.C.D} = (x + 2)(x - 3)$$

$(x - 3)$ divides $(x^2 - 4x + a)$ exactly

$$\Rightarrow (3)^2 - 4(3) + a = 0$$
$$\boldsymbol{a} = \textbf{3}$$

$(x + 2)$ divides $(x^2 + 4x + b)$ exactly

$$(-2)^2 + 4(-2) + b = 0$$
$$4 - 8 + b = 0$$
$$\textbf{b} = \textbf{4}$$