

$$\text{If } a + b = 1$$

$$a^2 + b^2 = 2$$

Then $a^8 + b^8 = ?$

$$b = 1 - a \text{ for,}$$

$$a^2 + (1 - a)^2 = 2 \Rightarrow 2a^2 - 2a - 1 = 0$$

$$\Rightarrow a^2 = a + \frac{1}{2}$$

$$\Rightarrow a^3 = a^2 + a + \frac{1}{4}$$

$$a^4 = a + \frac{1}{2} + a + \frac{1}{4}$$

$$a^4 = 2a + \frac{3}{4}$$

$$a^8 = 4a^2 + 3a + \frac{9}{16}$$

$$\text{or, } b^8 = 4b^2 + 3b + \frac{9}{16}$$

$$\begin{array}{r} + \\ \hline a^8 + b^8 = 4(a^2 + b^2) + 3(a + b) + \frac{9}{8} \end{array}$$

$$= 4 \cdot 2 + 3 \cdot 1 + \frac{9}{8}$$

$$a^8 + b^8 = \frac{97}{8}$$