

Minimize

$$3x^2 - 4xy + 5y^2 + 16x - 18y + 20$$

Local extreme values:

$$\left. \begin{aligned} f_x &= 6x - 4y + 16 = 0 \Rightarrow 3x - 2y = -8 \\ f_y &= -4x + 10y - 18 = 0 \Rightarrow 2x - 5y = -9 \end{aligned} \right\} \begin{aligned} x &= -2 \\ y &= 1 \end{aligned}$$

$$\left. \begin{aligned} f_{xx} &= 6 \\ f_{yy} &= 10 \\ f_{xy} &= -4 \end{aligned} \right\} \begin{aligned} D &= f_{xx} \cdot f_{yy} - f_{xy}^2 \\ D &= 6 \cdot 10 - (-4)^2 = 44 > 0 \end{aligned}$$

If $D > 0$, $P(-2, 1)$, $f_{xx} > 0 \rightarrow$ local Min.

$$\begin{aligned} P(-2, 1) &= 3(-2)^2 - 4(-2) \cdot 1 + 5 \cdot 1^2 + 16(-2) - 18 \cdot 1 + 20 \\ &\Rightarrow P(-2, 1) = -5 \rightarrow \text{Min. } \checkmark \end{aligned}$$