$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(z)\, dz, \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(z)\cos nz\, dz$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(z)\sin nz\, dz$$

$$f\left(\frac{lz}{\pi}\right) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nz + \sum_{n=1}^{\infty} b_n \sin nz$$

where $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f\left(\frac{lz}{\pi}\right) dz$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f\left(\frac{lz}{\pi}\right)\cos nz\, dz, \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f\left(\frac{lz}{\pi}\right)\sin nz\, dz$$

Now $x = \frac{lz}{\pi}, \; dx = \frac{l}{\pi}\, dz$

Thus $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$

where $a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\, dx, \quad a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right) dx$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right) dx.$$

Example 2.1.1. State the Euler's formulae when f(x) is expanded as a Fourier series in $c < x < c + 2\pi$.

Solution : The Fourier Series for $f(x)$ in the $c < x < c + 2\pi$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where $a_0 = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\, dx$ ... (1)

$a_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\cos nx\, dx$ ... (2)