$$\therefore\ B_n = \frac{2}{10}\int_0^{10} x\,(10-x)\,\sin\left(\frac{n\pi x}{10}\right) dx$$

$$= \frac{1}{5}\int_0^{10} (10x - x^2)\,\sin\left(\frac{n\pi x}{10}\right) dx$$

$$= \frac{1}{5}\left[(10x-x^2)\left[\frac{-\cos\frac{n\pi x}{10}}{\left(\frac{n\pi}{10}\right)}\right] - (10-2x)\left[\frac{-\sin\frac{n\pi x}{10}}{\left(\frac{n\pi}{10}\right)^2}\right] + (-2)\left[\frac{\cos\frac{n\pi x}{10}}{\left(\frac{n\pi}{10}\right)^3}\right]\right]_0^{10}$$

$$= \frac{1}{5}\left[-(10x-x^2)\left(\frac{10}{n\pi}\right)\cos\frac{n\pi x}{10} + (10-2x)\left(\frac{10}{n\pi}\right)^2 \sin\frac{n\pi x}{10} - 2\left(\frac{10}{n\pi}\right)^3 \cos\frac{n\pi x}{10}\right]_0^{10}$$

$$= \frac{1}{5}\left[\left(-0 + 0 - 2\left(\frac{10}{n\pi}\right)^3 \cos n\pi\right) - \left(-0 + 0 - 2\left(\frac{10}{n\pi}\right)^3\right)\right]$$

$$= \frac{1}{5}\left[-2\left(\frac{10}{n\pi}\right)^3 (-1)^n + 2\left(\frac{10}{n\pi}\right)^3\right]$$

$$= \frac{1}{5}\, 2\left(\frac{10}{n\pi}\right)^3 [1 - (-1)^n]$$

$$= \frac{2\,(1000)}{5n^3\pi^3}[1 - (-1)^n]$$

$$= \frac{400}{n^3\pi^3}[1 - (-1)^n]$$

$$= \frac{800}{n^3\pi^3} \text{ if } n \text{ is odd.}$$

$$= 0 \text{ if } n \text{ is even.}$$