Hence, the general solution is $z = \text{C.F.}$

$$= \phi_1 (y + x) + \phi_2 (y + 3x)$$

Example 1.5.2 : Solve $[D^2 - 2DD' + D'^2] z = 0$.

Solution : $[D^2 - 2DD' + D'^2] z = 0$.

The auxiliary equation is $m^2 - 2m + 1 = 0$

[Replace D by m and D$'$ by 1]

i.e., $(m - 1)^2 = 0$

$$m = 1, 1$$

Here, the roots are equal

$\therefore$ C.F. $= \phi_1 (y + x) + x \phi_2 (y + x)$.

Since R.H.S. is zero, there is no particular integral

Hence, the general solution is $z = \text{C.F}$

$$z = \phi_1 (y + x) + x \phi_2 (y + x)$$

Example 1.5.3 : Solve $[D^3 + DD'^2 - D^2 D' - D'^3] z = 0$

Solution : Given $[D^3 + DD'^2 - D^2 D' - D'^3] z = 0$

The auxiliary equation is $m^3 - m^2 + m - 1 = 0$

[Replace D by m and D$'$ by 1]

$$m^2 (m - 1) + (m - 1) = 0$$

$$(m - 1)(m^2 + 1) = 0$$

$$m = 1, m^2 + 1 = 0$$

i.e., $m = 1, m = \pm i$

i.e., $m = 1, m = i, m = -i$

Here, the roots are distinct

$\therefore$ C.F $= \phi_1 (y + x) + \phi_2 (y + ix) + \phi_3 (y - ix)$

Since R.H.S. is zero, there is no particular integral