$$q = \frac{\partial z}{\partial y} = 1 + f'(xy)\,x$$

$$q - 1 = xf'(xy) \qquad \dots (3)$$

$$\frac{(2)}{(3)} \Rightarrow \quad \frac{p-1}{q-1} = \frac{y}{x}$$

$$px - x = qy - y$$

$px - qy = x - y$ is the required p.d.e.

Example 1.2(a)(3) : Eliminate the arbitrary function f from $z = f\left(\frac{y}{x}\right)$ and form a partial differential equation.

[A.U. Trichy N/D 2009][A.U N/D 2012]

a.f
f
1
∴ we use p & q only

Solution : Given : $z = f\left(\frac{y}{x}\right) \qquad \dots (1)$

Differentiating (1) p.w.r. to x, we get

$$p = \frac{\partial z}{\partial x} = f'\left(\frac{y}{x}\right)\left(\frac{-y}{x^2}\right) \qquad \dots (2)$$

Differentiating (1) p.w.r. to y, we get

$$q = \frac{\partial z}{\partial y} = f'\left(\frac{y}{x}\right)\left(\frac{1}{x}\right) \qquad \dots (3)$$

$$\frac{(2)}{(3)} \Rightarrow \quad \frac{p}{q} = \frac{-y}{x}$$

Therefore $\qquad px = -qy$

i.e., $px + qy = 0$ is the required p.d.e.

Example 1.2(a)(4) : Form the p.d.e. by eliminating f from $z = f(x + y)$.

a.f
f
1
∴ we use p & q only

Solution : $z = f(x + y) \qquad \dots (1)$

Differentiating (1) p.w.r. to x, we get

$$p = \frac{\partial z}{\partial x} = f'(x + y) \qquad \dots (2)$$

Differentiating (1) p.w.r. to y, we get