$$= \frac{2}{l}\left[\frac{l^3}{3} - 0\right]$$

$$\text{(i.e.,)} \qquad a_0 = \frac{2}{3} l^2$$

$$a_n = \frac{2}{l}\int_0^l x^2 \cos\frac{n\pi x}{l}\, dx$$

| | |
|---|---|
| $u = x^2$ | $v = \cos\left(\frac{n\pi x}{l}\right)$ |
| $u' = 2x$ | $v_1 = \dfrac{\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)} = \frac{l}{n\pi}\sin\left(\frac{n\pi x}{l}\right)$ |
| $u'' = 2$ | $v_2 = \dfrac{-\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2} = -\left(\frac{l}{n\pi}\right)^2 \cos\left(\frac{n\pi x}{l}\right)$ |
| $u''' = 0$ | $v_3 = \dfrac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3} = -\left(\frac{l}{n\pi}\right)^3 \sin\left(\frac{n\pi x}{l}\right)$ |
| $\int uv\, dx = uv_1 - u' v_2 + u'' v_3 - \ldots$ | |



$$= \frac{2}{l}\left[x^2 \frac{l}{n\pi}\sin\frac{n\pi x}{l} + 2x\frac{l^2}{n^2\pi^2}\cos\frac{n\pi x}{l} - 2\frac{l^3}{n^3\pi^3}\sin\frac{n\pi x}{l}\right]_0^l$$

$$= \frac{2}{l}\left[\frac{2l^3}{n^2\pi^2}\right](-1)^n \qquad \boxed{a_n = \left[\frac{4l^2(-1)^n}{n^2\pi^2}\right]}$$