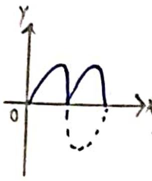
Cambridge A Level - Mathematics (9709) Topical summary = C1 - Algebra

[NKW]

Modulus functions

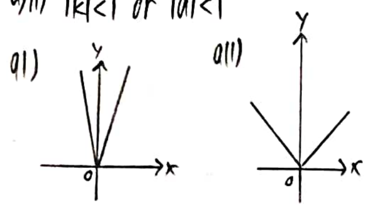
$y = |f(x)|$ = it is the obtained by reflecting the negative section of $y = f(x)$ in the x-axis.



Graphing $y = k|ax+b| + c$

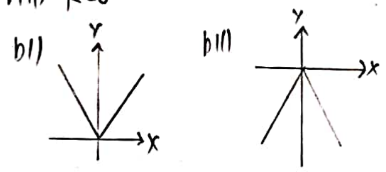
a)i) $|k| > 1$ or $|a| > 1$

a)ii) $|k| < 1$ or $|a| < 1$



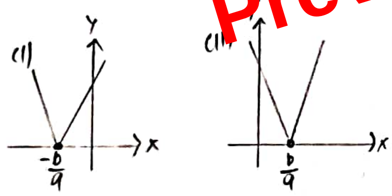
b)i) $k > 0$

b)ii) $k < 0$

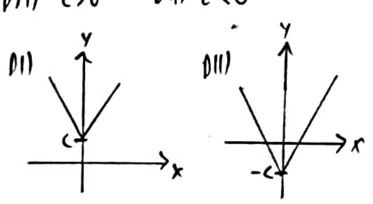


c)i) $x = \frac{b}{a}$, $\frac{b}{a} > 0$

c)ii) $x = \frac{b}{a}$, $\frac{b}{a} < 0$



D)i) $c > 0$ D)ii) $c < 0$



Solving modulus questions

1) Graphical method

↳ find points of intersection

2) Squaring methods

- only if both sides are positive for whole thing

$a^2 = b^2$

$a^2 - b^2 = 0$

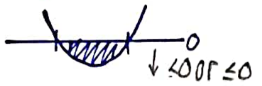
$(a+b)(a-b) = 0$

Solving inequalities involving modulus

1) Solve for

a)i) $|ax+b| < c$ OR $|ax+b| \le c$

Imagine this,



Answer:

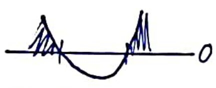
$-c < ax+b < c$

OR

$-c \le ax+b \le c$

a)ii) $|ax+b| > c$ OR $|ax+b| \ge c$

Imagine this,



Answer:

$ax+b \ge c$ & $ax+b \le -c$

$ax+b > c$ & $ax+b < -c$

Division of polynomials

- when a polynomial is divided by a divisor, $D(x)$

$\frac{p(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$

$Q(x)$ = quotient

$R(x)$ = remainder

$p(x) = D(x)Q(x) + R(x)$

Long division

ex = Divide $x^3 + 5x - 6$ by $x - 1$

$$\begin{array}{r} x^2 + x + 6 \\ x-1 \overline{) x^3 + 0x^2 + 5x - 6} \\ \ominus\ x^3 - x^2 \\ \hline x^2 + 5x \\ \ominus\ x^2 - x \\ \hline 6x - 6 \\ \ominus\ 6x - 6 \\ \hline 0 \end{array}$$

Synthetic Division

ex1 = Divide $x^4 - 2x^2 - 5x + 6$ by $(x-2)$ → $x = 2$

$$\begin{array}{c|ccccc} & 1 & 0 & -2 & -5 & 6 \\ 2 & \downarrow & 2 & 4 & 4 & -2 \\ \hline & 1 & 2 & 2 & -1 & 4 \\ & x^3 & x^2 & x & c & \end{array}$$

quotient $x^3 + 2x^2 + 2x - 1$

Remainder = 4

ex 2 = Divide $6x^3 + 4x^2 - x - 7$ by $(3x-4)$ → $x = \frac{4}{3}$ ← $\times 4$ ← $\times \frac{1}{3}$

$$\begin{array}{c|cccc} & 6 & 4 & -1 & -7 \\ 4 & \downarrow & 8 & 16 & 20 \\ \hline & 6 & 12 & 15 & 13 \\ \frac{1}{3} & 2 & 4 & 5 & \end{array}$$

$Q = 2x^2 + 4x + 5$

$R = 13$

ex 3 = Divide $x^4 + x^3 + 0x^2 + 9x + 7$ by $(x^2 - 2x + 4)$ → -4, → 2

$$\begin{array}{c|ccccc} & 1 & 1 & 0 & 9 & 7 \\ -4 & & -4 & -12 & -8 & \\ 2 & & 2 & 6 & & \\ \hline & 1 & 3 & 2 & 1 & -1 \end{array}$$

$Q = x^2 + 3x + 2$

$R = x - 1$

ex4 = Divide $6x^3 + 3x^2 - 8x + 4$ by $(3x^2 + 0x + 1)$ → -1, → 0, $\frac{1}{3}$

$$\begin{array}{c|cccc} & 6 & 3 & -9 & 4 \\ -1 & & & -2 & -1 \\ 0 & & 0 & 0 & \\ \hline & 6 & 3 & -11 & +3 \\ & \frac{1}{3} & \frac{2}{3} & 1 & \end{array}$$

$Q = 2x + 1$

$R = -11x + 3$

$p(x) = D(x)Q(x) + R(x)$

Remainder theorem

If $p(x)$ is divided by $ax+b$, then $p(x = -\frac{b}{a})$ will give you the remainder unless $ax+b$ is a factor, then there is no remainder.

Factor theorem

If $p(x)$ is divided by a factor of $ax+b$ then $p(x = -\frac{b}{a}) = 0$ as there won't be a remainder.

Factorisation

- Use inspection method.

Ex

If $p(x) = x^3 + 0x^2 - 4x - 15$ and $(x-3)$ is a factor of $p(x)$, factorise $p(x)$

$p(x) = (x-3)(x^2 + 3x + 5)$

x^2, $0x^2$, -15, ↳ quadratic factor

A factor of $f(x)$ and $f'(x)$

Sometimes, a factor can be for $f(x)$ and $f'(x)$ at the same time. Only appears if it shows up twice in $f(x)$.

Ex) If $(x-3)$ is a factor for $f(x)$ & $f'(x)$ how to show it?

$p(x) = 2x^3 + 11x^2 + 12x - 9$

$= (x-3)^2 (2x-1)$

$p'(x) = 2(x-3)(1)(2x-1) + (x-3)^2(2)$

$= 2(x-3)(2x-1) + 2(x-3)^2$

Partial fractions

① $\frac{3x}{(x+1)(2-x)} = \frac{A}{x+1} + \frac{B}{2-x}$

② $\frac{3x}{(x+1)(2-x)(x+2)} = \frac{A}{x+1} + \frac{B}{2-x} + \frac{C}{x+2}$

③ $\frac{3x}{(x+1)(4x^2-9)} = \frac{A}{x+1} + \frac{Bx+C}{4x^2-9}$

④ $\frac{3x}{x^2+6x+5} = \frac{3x}{(x+1)(x+5)}$

$= \frac{A}{x+1} + \frac{B}{x+5}$

⑤ $\frac{3x}{(x+1)(x-2)^2} = \frac{A}{x+1} + \frac{B}{(x-2)^2} + \frac{C}{x-2}$