How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed v0, the greater the range, as shown in Figure 3.40(a). The initial angle θ0 also has a dramatic effect on the range, as illustrated in Figure 3.40(b). For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with θ0=45º. This is true only for conditions neglecting air resistance. If air resistance is considered, the maximum angle is approximately 38º. Interestingly, for every initial angle except 45º, there are two angles that give the same range—the sum of those angles is 90º. The range also depends on the value of the acceleration of gravity g. The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range R of a projectile on level ground for which air resistance is negligible is given by

$$R = \frac{v_0^2 \sin 2\theta_0}{g}, \qquad \boxed{3.71}$$

where v0 is the initial speed and θ0 is the initial angle relative to the horizontal. The proof of this equation is left as an end-of-chapter problem (hints are given), but it does fit the major features of projectile range as described.

When we speak of the range of a projectile on level ground, we assume that R is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See Figure 3.41.) If the initial speed is great enough, the projectile goes into orbit. This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface. These and other aspects of orbital motion, such as the rotation of the Earth, will be covered analytically and in greater depth later in this text.

Once again, we see that thinking about one topic, such as the range of a projectile, can lead us to others, such as the Earth orbits. In Addition of Velocities, we will examine the addition of velocities, which is another important aspect of two-dimensional kinematics and will also yield insights beyond the immediate topic.

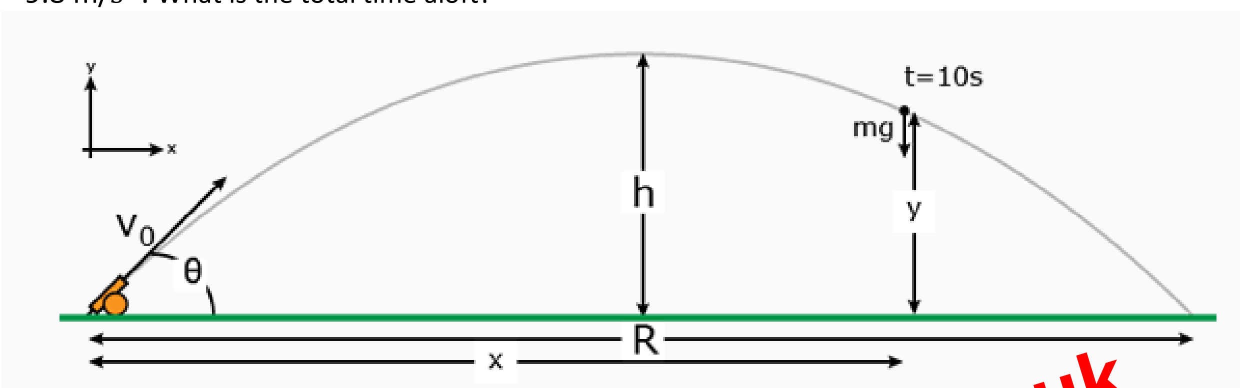
Figure 3.41 Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved.

$v_{hy} - v_{0y} = at$
$0 - v_{0y} = -9.8\ m/s^2 \cdot t$
$0 - 106.1\ m/s = -9.8\ m/s^2 \cdot t$
Solve for t
$t = \frac{-106.1\ m/s}{-9.8\ m/s^2}$
$t = 10.8\ s$
Now solve the first equation for h
$h = v_{0y}t + \frac{1}{2}at^2$
$h = (106.1\ m/s)(10.8\ s) + \frac{1}{2}(-9.8\ m/s^2)(10.8\ s)^2$
$h = 1145.9\ m - 571.5\ m$
$h = 574.4\ m$

The highest height the projectile reaches is 574.4 meters.

Problem 15.

A cannon is fired with muzzle velocity of 150 m/s at an angle of elevation = 45°. Gravity = 9.8 m/s^2. What is the total time aloft?



Let's set up what we know. First, let's define our variables.
v_0 = initial velocity = muzzle velocity = 150 m/s
v_x = horizontal velocity component
v_y = vertical velocity component
θ = angle of elevation = 45°
h = maximum height
R = range
x = horizontal position at t=10 s
y = vertical position at t=10 s
m = mass of projectile
g = acceleration due to gravity = 9.8 m/s^2

$t_{total} = t_{up} + t_{down}$

The same acceleration force acts on the projectile in both directions. The time down takes the same amount of time it took to go up.

$t_{up} = t_{down}$
or
$t_{total} = 2\ t_{up}$

we found t_{up} in Part a of the problem: 10.8 seconds
$t_{total} = 2\ (10.8\ s)$
$t_{total} = 21.6\ s$

The total time aloft for the projectile is 21.6 seconds.