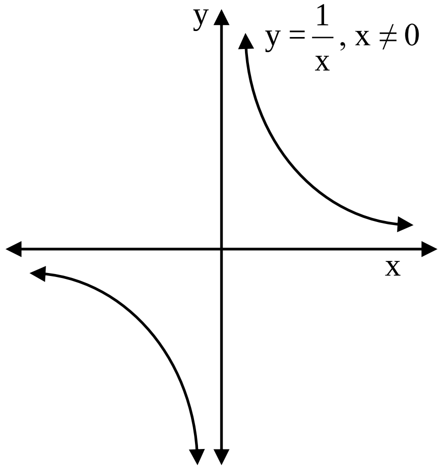
Fig. B



x	f(x)
0,01	100
0,001	1000
0,0001	10000
-0,01	-100
-0,001	-1000
-0,0001	-10000

As x approaches zero from the right, $\frac{1}{x}$ becomes positively infinite. As x approaches zero from the left, $\frac{1}{x}$ becomes negatively infinite. Symbolically we write

$$\lim_{x \to 0^+} \frac{1}{x} = \infty \qquad \text{and} \qquad \lim_{x \to 0^-} \frac{1}{x} = -\infty$$

Now let us again examine the function $f(x) = \frac{1}{x}$, $x \neq 0$, as x becomes infinite in both a positive and negative sense.

x	**f(x)**
1,000	.001
10,000	.0001
100,000	.00001
1,000,000	.000001

x	**f(x)**
-1,000	-.001
-10,000	-.0001
-100,000	-.00001
-1,000,000	-.000001

From the table above we can see that as x increases without bound through positive values, the corresponding values of f(x) approaches zero.
Like-wise, as x decreases without bound through negative values, the corresponding values of f(x) also approach zero. Symbolically we have

$$\lim_{x \to \infty} \frac{1}{x} = 0 \qquad \text{and} \qquad \lim_{x \to -\infty} \frac{1}{x} = 0$$

We can now turn to the evaluation of the limit of a quotient of two polynomials where the variable becomes infinite. For example consider

$$\lim_{x\to\infty}\frac{x^2+3x+5}{x^2-x+1}$$

It is clear that as $x \to \infty$, both numerator and denominator become infinite. However, the form of the quotient can be changed so that we can draw a conclusion as to whether or not a limit exists. Since $x \to \infty$, we are concerned only with those values of x which are very large. Thus, we can assume $x \neq 0$.

A frequently used "gimmick" is to divide both the numerator and denominator by the power of x which is the largest in either the numerator or denominator. In our example it is x^2. Thus

$$\lim_{x\to\infty}\frac{x^2+3x+5}{x^2-x+1} = \lim_{x\to\infty}\frac{x^2(1+\frac{3}{x}+\frac{5}{x^2})}{x^2(1-\frac{1}{x}+\frac{1}{x^2})}$$

$$= \frac{\lim\limits_{x\to\infty}1+3\lim\limits_{x\to\infty}\frac{1}{x}+5\lim\limits_{x\to\infty}\frac{1}{x^2}}{\lim\limits_{x\to\infty}1-\lim\limits_{x\to\infty}\frac{1}{x}+\lim\limits_{x\to\infty}\frac{1}{x^2}}$$

As $x \to \infty$, $\frac{1}{x}$ approaches 0. In fact, $\lim\limits_{x\to\infty}\frac{1}{x^p}=0$ for $p > 0$.

Thus, $\lim\limits_{x\to\infty}\frac{x^2+3x+5}{x^2-x+1} = \frac{1+3(0)+5(0)}{1-0+0} = \frac{1}{1} = 1$

Examples

1. $\lim\limits_{x\to\infty}\frac{8x^2+2x+3}{2x^3+3x-1} = \lim\limits_{x\to\infty}\frac{x^3\left(\frac{8}{x}+\frac{2}{x^2}+\frac{3}{x^3}\right)}{x^3\left(2+\frac{3}{x^2}-\frac{1}{x^3}\right)}$

$$= \frac{8\lim\limits_{x\to\infty}\frac{1}{x}+2\lim\limits_{x\to\infty}\frac{1}{x^2}+3\lim\limits_{x\to\infty}\frac{1}{x^3}}{\lim\limits_{x\to\infty}2+3\lim\limits_{x\to\infty}\frac{1}{x^2}-\lim\limits_{x\to\infty}\frac{1}{x^3}}$$

$$= \frac{8(0)+2(0)+3(0)}{2+3(0)-(0)}$$

$$= \frac{0}{2} = 0$$