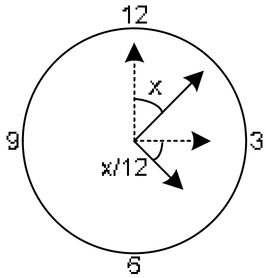
CLOCK PROBLEMS



where: **x** = distance traveled by the minute hand in minutes **x/12** = distance traveled by the hour hand in minutes

PROGRESSION PROBLEMS

a_1 = first term **a_n** = n^{th} term

a_m = any term before a_n **d** = common difference

S = sum of all "n" terms

ARITHMETIC PROGRESSION (AP)

- difference of any 2 no.'s is constant
- calcu function: **LINEAR (LIN)**

$$a_n = a_m + (n-m)d$$ n^{th} term

$$d = a_2 - a_1 = a_3 - a_2, ...etc$$ Common difference

$$S = \frac{n}{2}(a_1 + a_n)$$ Sum of ALL terms

$$S = \frac{n}{2}[2a_1 + (n-1)d]$$ Sum of ALL terms

GEOMETRIC PROGRESSION (GP)

- RATIO of any 2 adj, terms is always constant
- Calcu function: **EXPONENTIAL (EXP)**

$$a_n = a_m r^{n-m}$$ n^{th} term

$$r = \frac{a_2}{a_1} = \frac{a_3}{a_2}$$ ratio

$$S = \frac{a_1(r^n - 1)}{r - 1} \rightarrow r > 1$$ Sum of ALL terms, r >1

$$S = \frac{a_1(1 - r^n)}{1 - r} \rightarrow r < 1$$ Sum of ALL terms, r < 1

$$S = \frac{a_1}{1 - r} \rightarrow r < 1 \& n = \infty$$ Sum of ALL terms, r < 1 , n = ∞

HARMONIC PROGRESSION (HP)

- a sequence of number in which their reciprocals form an AP
- calcu function: **LINEAR (LIN)**

Mean – middle term or terms between two terms in the progression.

COIN PROBLEMS

Penny = 1 centavo coin **Nickel** = 5 centavo coin
Dime = 10 centavo coin
Quarter = 25 centavo coin
Half-Dollar = 50 centavo coin

DIOPHANTINE EQUATIONS

If the number of equations is less than the number of unknowns, then the equations are called "Diophantine Equations".

ALGEBRA 3

Fundamental Principle:
"If one event can occur in **m** different ways, and after it has occurred in any one of these ways, a second event can occur in **n** different ways, and then the number of ways the two events can occur in succession is **mn** different ways"

$$Q = 1 - P$$

MULTIPLE EVENTS

Mutually exclusive events without a common outcome

$$P_{A \text{ or } B} = P_A + P_B$$

Mutually exclusive events with a common outcome

$$P_{A \text{ or } B} = P_A + P_B - P_{A\&B}$$

Dependent/Independent Probability

$$P_{AandB} = P_A \times P_B$$

REPEATED TRIAL PROBABILITY

$$P = nC_r \, p^r \, q^{n-r}$$

p = probability that the event happen **q** = probability that the event failed

VENN DIAGRAMS

Venn diagram in mathematics is a diagram representing a set or sets and the logical, relationships between them. The sets are drawn as circles. The method is named after the British mathematician and logician John Venn.

PLANE TRIGONOMETRY

ANGLE, MEASUREMENTS & CONVERSIONS

1 revolution = 360 degrees

1 revolution = 2π radians

1 revolution = 400 grads

1 revolution = 6400 mils

1 revolution = 6400 gons

Relations between two angles (A & B)

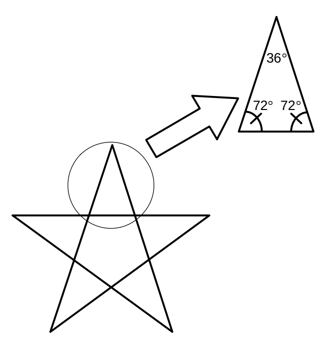
Complementary angles → **A + B = 90°**

Supplementary angles → **A + B = 180°**

Explementary angles → **A + B = 360°**

Angle (θ)	Measurement
NULL	$\theta = 0°$
ACUTE	$0° < \theta < 90°$
RIGHT	$\theta = 90°$
OBTUSE	$90° < \theta < 180°$
STRAIGHT	$\theta = 180°$
REFLEX	$180° < \theta < 360°$
FULL OR PERIGON	$\theta = 360°$

Pentagram – golden triangle (isosceles)



TRIGONOMETRIC IDENTITIES

$$0° < a + b + c < 360°$$

5. **The sum of the angles** of a spherical triangle is **greater that 180° and less than 540°**.

$$180° < A + B + C < 540°$$

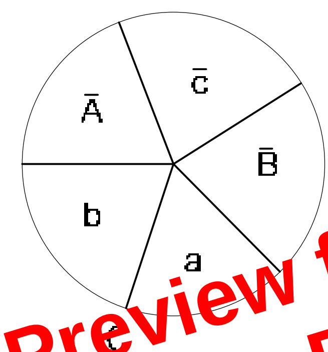
6. The **sum of any two angles** of a spherical triangle is **less than 180° plus the third angle**.

$$A + B < 180° + C$$

SOLUTION TO RIGHT TRIANGLES

NAPIER CIRCLE

Sometimes called Neper's circle or Neper's pentagon, is a mnemonic aid to easily find all relations between the angles and sides in a right spherical triangle.



Napier's Rules

1. The sine of any middle part is equal to the product of the cosines of the opposite parts.
   Co-op

2. The sine of any middle part is equal to the product of the tangent of the adjacent parts.
   Tan-ad

Important Rules:

1. In a right spherical triangle and oblique angle and the side opposite are of the same quadrant.

2. When the hypotenuse of a right spherical triangle is less than 90°, the two legs are of the same quadrant and conversely.

3. When the hypotenuse of a right spherical triangle is greater than 90°, one leg is of the first quadrant and the other of the second and conversely.

QUADRANTAL TRIANGLE

is a spherical triangle having a side equal to 90°.

SOLUTION TO OBLIQUE TRIANGLES

Law of Sines:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Law of Cosines for sides:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

Law of Cosines for angles:

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a$$

$$\cos B = -\cos A \cos C + \sin A \sin C \cos b$$

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

AREA OF SPHERICAL TRIANGLE

$$A = \frac{\pi R^2 E}{180°}$$

R = radius of the sphere
E = spherical excess in degrees,

$$E = A + B + C - 180°$$

Total length of cable = $S_1 + S_2$

$$V^2 = V_0^2 \pm 2aS$$

Even elevation of supports + (sign) = body is speeding up – (sign) = body is slowing down

$T = wy$

Constant Acceleration: Vertical Motion

$H = wc$ $\pm H = V_0 t - \frac{1}{2} gt^2$ $y^2 = S^2 + c^2$

$x = c \ln \left(\frac{S + y}{c} \right)$

$$V = V_0 \pm gt$$

$V^2 = V_0^2 \pm 2gH$ $Span = 2x$

Total length of cable = 2S

+ (sign) = body is moving down
– (sign) = body is moving up

Values of g,

	SI (m/s²)	English (ft/s²)
general	9.81	32.2
estimate	9.8	32
exact	9.806	32.16

Variable Acceleration $V = \frac{dS}{dt}$ $a = \frac{dV}{dt}$

ROTATION (PLANE MOTION)

Relationships between linear & angular parameters:

$$V = r\omega$$

$$a = r\alpha$$

PROJECTILE MOTION

V = linear velocity
ω = angular velocity (rad/s)

about which one knows nothing other than the balance of energy and matter transfer.

ZEROTH LAW OF THERMODYNAMICS

stating that thermodynamic equilibrium is an equivalence relation.

If two thermodynamic systems are in thermal equilibrium with a third, they are also in thermal equilibrium with each other.

FIRST LAW OF THERMODYNAMICS

about the conservation of energy

The increase in the energy of a closed system is equal to the amount of energy added to the system by heating, minus the amount lost in the form of work done by the system on its surroundings.

SECOND LAW OF THERMODYNAMICS

about entropy

The total entropy of any isolated thermodynamic system tends to increase over time, approaching a maximum value.

THIRD LAW OF THERMODYNAMICS,

about absolute zero temperature

As a system asymptotically approaches absolute zero of temperature all processes virtually cease and the entropy of the system asymptotically approaches a minimum value. This law is more clearly stated as: "the entropy of a perfectly crystalline body at absolute zero temperature is zero."

STRENGTH OF MATERIALS

SIMPLE STRESS

$$Stress = \frac{Force}{Area}$$

Axial Stress

the stress developed under the action of the force acting axially (or passing the centroid) of the resisting area.

$$\sigma_{axial} = \frac{P_{axial}}{A}$$

P_{axial} ⊥ Area

σ_{axial} = axial/tensile/compressive stress **P** = applied force/load at centroid of x'sectional area
A = resisting area (perpendicular area)

Shearing stress

the stress developed when the force is applied parallel to the resisting area.

$$\sigma_s = \frac{P}{A}$$

$P_{applied}$ ∥ Area

σ_s = shearing stress **P** =
applied force or load
A = resisting area (sheared area)

Bearing stress

the stress developed in the area of contact (projected area) between two bodies.

C_m = book value **C_n** = salvage or scrap value
n = life of the property **D_m** = total depreciation after m-years **m** = m^th year

Sinking Fund Method (SFM)

$$d = \frac{(C_0 - C_n)i}{(1+i)^n - 1}$$

$$D_m = \frac{d[(1+i)^m - 1]}{i}$$

$C_m = C_0 - D_m$

i = standard rate of interest

Sum of the Years Digit (SYD) Method

$$d_m = (C_0 - C_n)\left[\frac{2(n-m+1)}{n(n+1)}\right]$$

$$D_m = (C_0 - C_n)\left[\frac{m(2n-m+1)}{n(n+1)}\right]$$

$$SYD = \frac{n(n+1)}{2}$$

$C_m = C_0 - D_m$

SYD = sum of the years digit **d_m** = depreciation at year m

Declining Balance Method (DBM)

C^n

$$k = 1 - \sqrt[n]{\frac{C_n}{C_o}}$$

Matheson Formula

$$k = 1 - \sqrt[m]{\frac{C_m}{C_o}}$$

$$C_m = C_0(1-k)^m$$

$$d_m = kC_0(1-k)^{m-1}$$

$D_m = C_0 - C_m$

k = constant rate of depreciation

CAPITALIZED AND ANNUAL COSTS

$$CC = C_0 + P$$

CC = Capitalized Cost
C_0 = first cost
P = cost of perpetual maintenance (A/i)

$$AC = d + C_0(i) + OMC$$

AC = Annual Cost **d** = Annual depreciation cost
i = interest rate
OMC = Annual operating & maintenance cost

BONDS

$$P = P_{anuity} + P_{cpd\ interest}$$

$$P = Zr\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] + \frac{C}{(1+i)^n}$$

P = present value of the bond **Z** = par value or face value of the bond **r** = rate of interest on the bond per period **Z_r** = periodic dividend **i** = standard interest rate **n** = number of years before redemption