$\Rightarrow \qquad p\left[x^2p^2+yp+x^2y^2p+y^3\right]=0$

$\Rightarrow \qquad p\left[x^2p\left(p+y^2\right)+y\left(p+y^2\right)\right]=0$

$\Rightarrow \qquad p\left(p+y^2\right)\left(x^2p+y\right)=0$

$\therefore\ \ p=0,\ p+y^2=0$ and $x^2p+y=0$

Now, $\quad p=0$

$\Rightarrow \qquad \frac{dy}{dx}=0$

Integrating on both sides, we get

$y=c_1 \Rightarrow y-c_1=0$ ... (1)

Now, $\quad p+y^2=0$

$\Rightarrow \qquad \frac{dy}{dx}=-y^2$

$\Rightarrow \qquad -\frac{dy}{y^2}=dx$

Integrating on both sides, we get

$\frac{1}{y}=x+c_2 \quad \Rightarrow \frac{1}{y}-x-c_2=0$ ... (2)

and $\qquad x^2p+y=0$

$\Rightarrow \qquad x^2\frac{dy}{dx}+y=0$

$\Rightarrow \qquad \frac{dy}{y}=-\frac{dx}{x^2}$

Integrating on both sides, we get

$\log y=\frac{1}{x}+c_3$

$\Rightarrow \log y-\frac{1}{x}-c_3=0$ ... (3)

The required solution is,

$\boxed{(y-c)\left(\frac{1}{y}-x-c\right)\left(\log y-\frac{1}{x}-c\right)=0}$ where $c_1=c_2=c_3=c$ **Answer**

Problem 3 **Solve:** $x^2\left(\frac{dy}{dx}\right)^2+xy\frac{dy}{dx}-6y^2=0$ **[RGPV Dec. 2020]**

Solution: Given, $x^2\left(\frac{dy}{dx}\right)^2+xy\frac{dy}{dx}-6y^2=0$

Taking, $p = \dfrac{dy}{dx}$

$$\therefore \quad x^2p^2 + xyp - 6y^2 = 0 \qquad \ldots(1)$$

$$x^2p^2 + 3xyp - 2xyp - 6y^2 = 0$$

$$\Rightarrow \quad xp(px+3y) - 2y(px+3y) = 0$$

$$\Rightarrow \quad (px+3y)(px-2y) = 0$$

$$\therefore \quad px + 3y = 0$$

$$\Rightarrow \quad \frac{dy}{y} = -3\frac{dx}{x}$$

$$\Rightarrow \quad \log y = -3\log x + \log c_1$$

$$\Rightarrow \quad \log y = \log\left(\frac{c_1}{x^3}\right)$$

$$\Rightarrow \quad y = \frac{c_1}{x^3}$$

$$\Rightarrow \quad x^3y - c_1 = 0 \qquad \ldots(2)$$

$$px - 2y = 0$$

$$\Rightarrow \quad x\frac{dy}{dx} - 2y = 0$$

$$\Rightarrow \quad \frac{dy}{y} = 2\frac{dx}{x}$$

$$\Rightarrow \quad \log y = 2\log x + \log c_2$$

$$\Rightarrow \quad \log y = \log\left(c_2x^2\right)$$

$$\Rightarrow \quad y = x^2c_2$$

$$\Rightarrow \quad yx^{-2} - c_2 = 0 \qquad \ldots(3)$$

The required solution is,

$$\boxed{\left(x^3y - c\right)\left(yx^{-2} - c\right) = 0} \qquad \text{Answer}$$

IMPROVE YOUR SELF

Solve the following differential Equations:

1. $p^2 - 7p + 12 = 0$

   [Answer: $(y-4x-c)(y-3x-c) = 0$ **]**

2. **Solve:** $p^2 + 2py\cot x - y^2 = 0$ **[RGPV Dec. 2015]**

   [Answer: $\left[y(1+\cos x) - c\right]\left[y(1-\cos x) - c\right] = 0$ **]**

3. **Solve:** $x^2\left(\dfrac{dy}{dx}\right)^2 + xy\dfrac{dy}{dx} - 6y^2 = 0$

   [Answer: $\left(x^3y - c\right)\left(yx^{-2} - c\right) = 0$ **]**

Method 2 [Solvable by x]

Concept: The differential equation said to be solvable for x when x can be separate in term of y such that

$$x = f(y, p)$$

Problem 1 **Solve:** $y = 2px + y^2 p^3$ **[RGPV Dec. 2019]**

Solution: Given differential Equation is,

$$y = 2px + y^2 p^3 \qquad \ldots (1)$$

Clearly it is solvable for x, then

$$2x = \frac{y}{p} - y^2 . p^2$$

Differentiate w.r.t. y, we get

$$2\frac{dx}{dy} = \frac{p.1 - y.\frac{dp}{dy}}{p^2} - \left[y^2 2p\frac{dp}{dy} + p^2 2y \right]$$

$$\Rightarrow \qquad \frac{2}{p} = \frac{1}{p} - \frac{y}{p^2}\frac{dp}{dy} - 2py^2\frac{dp}{dy} - 2p^2 y$$

$$\Rightarrow \qquad \frac{1}{p} + 2p^2 y = -\frac{dp}{dy}\left[\frac{y}{p^2} + 2py^2 \right]$$

$$\Rightarrow \qquad \frac{1}{p}\left[1 + 2p^3 y \right] = -\frac{dp}{dy}\frac{y}{p^2}\left[1 + 2p^3 y \right]$$

$$\Rightarrow \qquad 1 = -\frac{dp}{dy}\frac{y}{p}$$

$$\Rightarrow \qquad \frac{dp}{p} = -\frac{dy}{y}$$

Integrating both sides, we get

$$\log p = -\log y + \log c$$

$$\Rightarrow \qquad \log p = \log\left(\frac{c}{y} \right)$$

$$\Rightarrow \qquad p = \frac{c}{y}$$

Putting the value of p in equation (1), we get

$$y = 2\left(\frac{c}{y} \right)x + y^2 \left(\frac{c}{y} \right)^3$$

$$\Rightarrow \qquad y^2 = 2c + c^3 \qquad \textbf{Answer}$$

Problem 2 **Solve:** $y^2 \log y = x\,y\,p + p^2$ **[RGPV June 2015]**

Solution: Given differential equation is

[Answer: $y(c-1)=cx(c-1)+c$ **]**

Video Lectures Links			
S.No.	**Lecture No**	**Topic**	**Link**
1	Lecture 0	Differential Equation-Ordinary Differential Equation	https://youtu.be/GtZnXWdTD5Q
2	Lecture 1	Ordinary Differential Equation-Formation of Differential Equation	https://youtu.be/nyKypjThv0c
3	Lecture 2	Ordinary Differential Equation-Variable Separable form	https://youtu.be/C_2ScUovh8Y
4	Lecture 3	Ordinary Differential Equation-Homogeneous Differential Equation of first order	https://youtu.be/uJMDxnr2B8s
5	Lecture 4	Ordinary Differential Equation-Non-Homogeneous Differential Equation of First order	https://youtu.be/8cIHETZeuK
6	Lecture 5	Ordinary Differential Equation-Linear Differential Equation of First order	https://youtu.be/bJ1TqnXV_D8
7	Lecture 6	Ordinary Differential Equation-Bernoulli's Equation of First order	https://youtu.be/ua1Y_awO0Bo
8	Lecture 7	Ordinary Differential Equation-Exact differential Equation of First order and First Deg	https://youtu.be/DkztYqeADIE
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Method 1: When Q = e^{ax}

(i). $\dfrac{1}{f(D)}e^{ax} = \dfrac{1}{f(a)}e^{ax}$, when $f(a) \neq 0$

(ii). $\dfrac{1}{f(D)}e^{ax} = e^{ax}\left[\dfrac{1}{f(D+a)}.1\right]$, when $f(a) = 0$

(iii). $\dfrac{1}{f(D)}e^{ax} = \left[\dfrac{x}{\dfrac{\partial}{\partial D}f(D)}\right]e^{ax}$, when $f(a) = 0$

(iv). $\dfrac{1}{f(D)}e^{ax} = \left[\dfrac{x^n}{\dfrac{\partial^n}{\partial D^n}f(D)}\right]e^{ax}$, when $f(a) = 0$

Problem 1 Solve: $\left(D^2+3D+5\right)y = e^{2x}$

Solution: Given, $\left(D^2+3D+5\right)y = e^{2x}$ … (1)

The A.E. equation is,

$$m^2+3m+5=0$$

$\Rightarrow$ $$m = \frac{-3 \pm \sqrt{3^2-4\times1\times5}}{2\times1} = \frac{-3\pm\sqrt{9-20}}{2}$$

$\Rightarrow$ $$m = -\frac{3}{2} \pm \frac{i\sqrt{11}}{2}$$

$\therefore$ $$C.F. = e^{-(3x/2)}\left[c_1\cos\left(\frac{\sqrt{11}}{2}x\right)+c_2\sin\left(\frac{\sqrt{11}}{2}x\right)\right]$$

Now, $$P.I. = \frac{1}{D^2+3D+5}e^{2x} = \frac{1}{2^2+3(2)+5}e^{2x}$$

$\Rightarrow$ $$P.I. = \frac{e^{2x}}{15}$$

The required solution is,

$$y = C.F. + P.I.$$

$\Rightarrow$ $$\boxed{y = e^{-(3x/2)}\left[c_1\cos\left(\frac{\sqrt{11}}{2}x\right)+c_2\sin\left(\frac{\sqrt{11}}{2}x\right)\right]+\frac{e^{2x}}{15}}$$ **Answer**

Problem 2 Solve: $\left(D^2-3D+2\right)y = e^{2x}$

Solution: The A.E. is

$$y=(c_1+xc_2)\,e^{3x}+\frac{1}{9}\left[x^2+\frac{4x}{3}+\frac{2}{3}\right]+2e^{2x}$$ **Answer**

IMPROVE YOUR SELF

Solve the following differential Equations:

1. $(D^2+4)\,y=x^2$ **[Answer:** $y=c_1\cos 2x+c_2\sin 2x+\frac{1}{4}\left(x^2-\frac{1}{2}\right)$ **]**

2. $\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}+2\frac{dy}{dx}=x^2$ **[Answer:** $y=c_1+c_2e^{-2x}+c_3e^{-x}+\frac{1}{12}\left[2x^3-9x^2+21x\right]$ **]**

3. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=2x+x^2$ **[Answer:** $y=(c_1+xc_2)e^{-x}+x^2-2x+2$ **]**

Lecture-11

Method 3: When Q = Sin*ax* or Cos*ax*

(i) . $\frac{1}{f(D^2)}\sin ax \text{ or } \cos ax=\frac{1}{f(-a^2)}\sin ax \text{ or } \cos ax$, When $f(-a^2)\neq 0$

(ii). $\frac{1}{D^2+a^2}\sin ax=-\frac{x}{2a}\cos ax$, When $f(-a^2)=0$

(iii). $\frac{1}{D^2+a^2}\cos ax=\frac{x}{2a}\sin ax$, When $f(-a^2)=0$

Problem 1 **Solve the differential equation**

$$(D^2+D+1)\,y=\sin x$$

Solution: Given differential equation is

$$(D^2+D+1)\,y=\sin x\text{, Here, } D\equiv\frac{d}{dx}$$

The A.E. is,

$$m^2+m+1=0$$

$$\Rightarrow\quad m=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1\pm\sqrt{3}\,i}{2}=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$$

$$\therefore\ C.F.=e^{-x/2}\left[c_1\cos\left(\frac{\sqrt{3}}{2}x\right)+c_2\sin\left(\frac{\sqrt{3}}{2}x\right)\right]$$

Now, $$P.I.=\frac{1}{D^2+D+1}\sin x=\frac{1}{-1^2+D+1}\sin x$$

$$=\frac{1}{D}\sin x=-\cos x$$

$$\boxed{y=\left(c_1+xc_2\right)e^{-x}+\frac{x}{2}\left(\sin x\right)+\frac{1}{2}\left[-\sin x+\cos x\right]}$$ **Answer**

IMPROVE YOUR SELF

Solve the following differential Equations:

1. $\dfrac{d^2y}{dx^2}-3\dfrac{dy}{dx}+2y=xe^x$ **[Answer:** $y=c_1e^x+c_2e^{2x}-e^x\left(\dfrac{x^2}{2}+x\right)$ **]**

2. $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y=xe^x\sin x$ **[Answer:** $y=\left(c_1+xc_2\right)e^x-e^x\left[x\sin x+2\cos x\right]$ **]**

3. $(D^2-4D+3)y=2xe^x+3e^x\sin x$ **[Answer:** $y=c_1e^x+c_2e^{3x}-\dfrac{e^x}{2}\left(x^2+x\right)+\dfrac{3}{5}e^x\left(2\cos x-\sin x\right)$ **]**

Method 6: Special Condition

(i). $\dfrac{1}{f(D)}x^m\sin ax=I.P.\left[\dfrac{1}{f(D)}\right]x^m(\cos ax+i\sin ax)=I.P.\left[\dfrac{1}{f(D)}\right]x^m e^{aix}$

(ii). $\dfrac{1}{f(D)}x^m\cos ax=R.P.\left[\dfrac{1}{f(D)}\right]x^m(\cos ax+i\sin ax)=R.P.\left[\dfrac{1}{f(D)}\right]x^m e^{aix}$

Where $m\geq 1$ and m is positive integer.

(iii). $\dfrac{1}{D-\alpha}Q=e^{\alpha x}\int e^{-\alpha x}Q\,dx$

(iv). $\dfrac{1}{D+\alpha}Q=e^{-\alpha x}\int e^{\alpha x}Q\,dx$

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