GRE

Arithmetic

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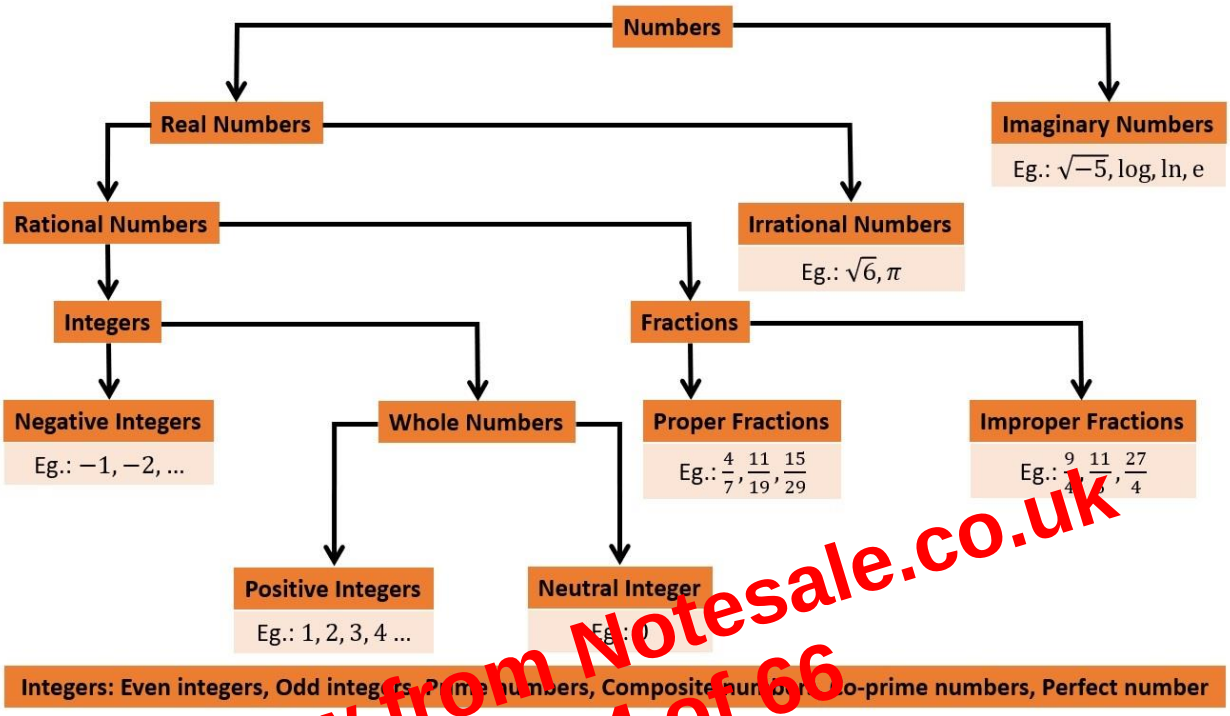
Classification of Numbers

The numbers are classified into the following ways:



Numbers are classified to real numbers and imaginary numbers. Real numbers are the numbers which can be represented on number line, and imaginary numbers are the numbers which can't represented on number line. Few of examples of imaginary numbers are shown. Imaginary numbers are not a part of GRE quantitative reasoning.

Real numbers are further classified to rational numbers and irrational numbers. Rational numbers are the numbers which can be represented in form of ratio, and irrational numbers are the numbers which can't represented in form of ratio. $\sqrt{6}$ is an irrational number because it can't be written in form of ratio. Similarly, π is also an irrational number because it can't be written in form of ratio. 22/7 is an approximate value of π.

Rational numbers are further classified to integers and fractions. Integers don't have fractional component. Integers are classified to negative integers and whole numbers. Negative integers are the integers which less than 0. Examples of negative integers are -1, -2, -3, up to infinite.

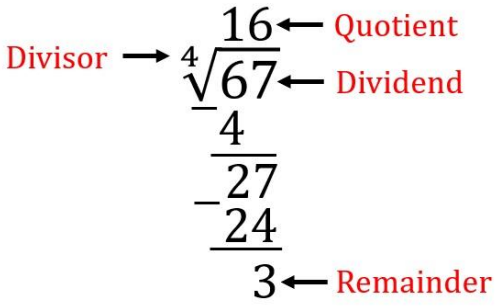
Whole numbers are classified to positive integers and neutral integer. Positive integers are the integers which are more than 0. Example of positive integers are 1, 2, 3, 4, up to infinite. Neutral integer is integer which is neither positive nor negative. Zero is a neutral integer.

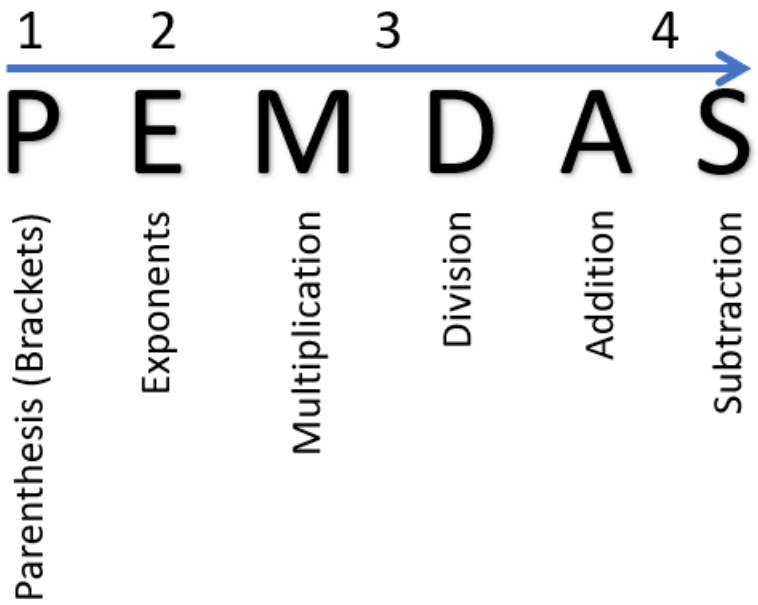
Fractions are the numbers which are represented in form of ratio of numerator and denominator. If numerator is less than denominator, the fractions are called proper fractions. Some examples are shown. If numerator is more than denominator, the fractions are called improper fractions. Some examples of improper fractions are as shown.

Few more integers are even integers and odd integers. Even integers are the integers which are multiples of 2. Some examples of even integers are 0, 2, 4, -2, -4, -6, etc. Odd integers are the integers which are divided by 2 and left 1 as a reminder. 9, 7, 11, -5, -3, -1, etc. are odd integers.

Prime number is a number which has only two factors; one and number itself. No negative numbers are prime. 1 is not a prime number. 2 is only even prime and it is the smallest prime. Few examples of prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, ... Composite numbers are the numbers which have factors other than one and number itself. 16 is a composite number because 16 has factors 2, 4, and 8, other than 1 and 16. Coprime numbers are the numbers that have only one factor is common to them, namely 1. For example, 16 and 31 are coprime numbers because they have only one common factor, which is 1 only. A perfect number is a positive integer that, excluding the number itself, is equal to the sum of its positive factors. For example, 6 has factors 1, 2, 3, and 6. By adding factors of 6 other than number itself, sum is equal to 6.

Division Rule





- **Multiplication and division:** The multiplication and division have the same priority, but in left to right direction whichever comes first, that should be solved first. If left to right direction, multiplication comes first, multiplication performs the first, and in left to right direction, division comes first, division performs the first.
- **Addition and subtraction:** The addition and subtraction have the same priority, but in left to right direction whichever comes first, that should be solved first. In the left to right direction, addition comes first, addition performs the first, and in left to right direction, subtraction comes first, subtraction performs the first.

Example: Solve [6 x 3 + 9 – (4^2 x 3) ÷ 4] x 2.

Solution:

[6 x 3 + 9 – (4^2 x 3) ÷ 4] x 2	
[6 x 3 + 9 – (16 x 3) ÷ 4] x 2	(Exponent solved)
[6 x 3 + 9 – 48 ÷ 4] x 2	(Multiplication inside the bracket)
[18 + 9 – 48 ÷ 4] x 2	(Multiplication first, solved multiplication)
[18 + 9 – 12] x 2	(Division has higher priority)
[27 – 12] x 2	(Addition has higher priority)
15 x 2 = 7.5	
Answer = 7.5	

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Consecutive Integers

A sequence of integers increasing (or decreasing) by 1 is called consecutive integers.

Example: 5, 6, 7, 8, 9, 10 are consecutive increasing integers, 12, 11, 10, 9, 8, 7, 6 are consecutive decreasing integers.

Consecutive increasing integers are written x, x + 1, x + 2, x + 3, x + 4, … in form of variables. Consecutive decreasing integers are written x, x – 1, x – 2, x – 3, x – 4, … in form of variables.

Example: The sum of five consecutive numbers is 55. What is the highest of the five integers?

- O 9
- O 10
- O 11
- O 12
- O 13

Solution:

Assume the smallest integer among five is x. So, sum is x + (x + 1) + (x + 2) + (x + 3) + (x + 4) = 55.

5x + 10 = 55; 5x = 45; x = 9. Numbers are 9, 10, 11, 12, and 13. The greatest among them is 13. The answer is 13.

Number Theory

Factors

Factors of any number are the numbers which can divide that number perfectly.

Example: Factors of 120 are 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, and 120.

Similarly factors of 100 are 1, 2, 5, 10, 20, 25, 50, and 100.

Factors of each number are finite.

Multiples

A number that can be equally divided by any natural number is called a multiple of that number. That is, no leftovers are created as a result of this technique. A number's multiples are created by multiplying it by any natural integer, such as 1, 2, 3, 4, …

O 2

O 3

21. A certain theater has 100 balcony seats. For every $2 increase in the price of a balcony seat above $10, 5 fewer seats will be sold. If all the balcony seats are sold when the price of each seat is $10, which of the following could be the price of a balcony seat if the revenue from the sale of balcony seats is $1,360? [24]

O $12

O $14

O $16

O $17

O $18

22. In writing all of the integers from 1 to 300, how many times is the digit 1 used? [25]

23. Carla has 1/4 more sweaters than cardigans, and 2/5 fewer cardigans than turtle necks. If she has at least one of each item, what is the minimum total number of turtlenecks plus sweaters that Carla could have? [26]

24. The first term in a certain sequence is 1, the 2nd term in the sequence is 2, and, for all integers n ≥ 3, the nth term in the sequence is the average (arithmetic mean) of the first n – 1 term in the sequence. What is the value of the 6th term in the sequence? [27]

25. What is the remainder when $13^{17} + 17^{13}$ is divided by 10? [28]

26. If x, y, and z are positive numbers such that 3x < 2y < 4z, which of the following statements could be true? [29]

Indicate all such statements.

☐ x = y

☐ y = z

Answers of Practice Questions

1.	B	2.	B	3.	D	4.	C
5.	E	6.	D	7.	E	8.	B
9.	B	10.	C	11.	D	12.	C
13.	A	14.	C	15.	D	16.	D
17.	B	18.	D	19.	C	20.	D
21.	C	22.	160	23.	35	24.	1.5
25.	0	26.	B, C, D	27.	1, 3, 7, 9	28.	$-10, 2^{-4}$
29.	B, C	30.	B	31.	C	32.	D
33.	C	34.	A	35.	D	36.	D
37.	A	38.	B	39.	B	40.	A
41.	A	42.	A	43.	B	44.	D
45.	C	46.	B	47.	D	48.	C
49.	D	50.	A	51.	C	52.	E
53.	C	54.	C	55.	C	56.	D

Solution

1 Let's take K = 2,

Quantity – A: (-2) * 1/2 = -1

Quantity – B: 2 * 1/2 = 1

Answer is B.

Let's take K = -2,

Quantity – A: 2 * 1/(-2) = -1

Quantity – B: (-2) * 1/(-2) = 1

Answer is B.

Hence, the answer is D.

2 p, q, and r are single digit positive different integers and r/q = p.

If we assume r = 9, 9/3 = 3 not possible because q = p.

Assume r = 8, 8/2 = 4 or 8/4 = 2. This is possible and the maximum value of r = 8.

Hence, the answer is B.

3 Let's assume c = 8

Quantity – A: Divisors of 8 are 1, 2, 4, 8. Quantity – A = 2 * 4 = 8

Quantity – B: No. of positive divisors of 2 * 8 = 16 are 1, 2, 4, 8, 16, and they are 5.

Right now, answer is A.

Let's assume c = 15

Quantity – A: Divisors of 15 are 1, 3, 5, 15. Quantity – A = 2 * 4 = 8

Quantity – B: No. of positive divisors of 2 * 15 = 30 are 1, 2, 3, 5, 6, 10, 15, 30 and they are 8.

Now, answer is C.

Hence, the answer is D.

4 Original ratio of pennies to nickels is 7:4. P/N = 7/4,

After three pennies removed, new ratio is 3:2, (P – 3)/N = 3/2

Solving for Nickel, N = 12.

Hence, the answer is C.

5 $p^2 \leq 1$; $-1 \leq p \leq 1$

So, we get:

Quantity - A: (-1) + (-1) + (-1) = (-3)

Quantity - B: (-1) + 1 + (-1) + 1 = 0

In this case, quantity B is greater.

Case ii: x is EVEN

If x is even, then x^2 is even, x^3 is even, and x^4 is even.

Also, if x is EVEN, then 2x is even, 3x is even and 4x is even.

So, we get:

Quantity – A: 1 + 1 + 1 = 3

Quantity – B: (-1) + 1 + (-1) + 1 = 0

In this case, Quantity B is greater.

In both possible cases, Quantity B is greater.

Hence, the answer is B. [87]

44 Let's test some possible values of x, y and z.

Case i: Since x > z and y > z, one possible set of values is x = 1, y = 1, and z = 0

We get:

Quantity A: x + y = 1 + 1 = 2

Quantity B: 0

In this case, Quantity A is greater.

Case ii: Since x > z and y > z, one possible set of values is x=−1, y=−1, and z=−2

We get:

Quantity A: x + y = (−1) + (−1) = −2

Quantity B: −2

In this case, the two quantities are equal

Hence, the answer is D. [88]

45 If you put the sequence as 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, you can see that it conforms to the condition that each term after the first is the average of the preceding term and the following term:

$a_{10} - a_8 = 10 - 8 = 2$

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