Advantages and disadvantages of angle-axis

- Angle-axis is minimal orientation representation.
  - 4-parameters are needed
- Orientations represented in angle-axis cannot be *added* together easily.
  - We can neither multiply nor add rotation axis or rotation angle.
  - We should convert the angle-axis rotations to rotation matrices, then multiply them.

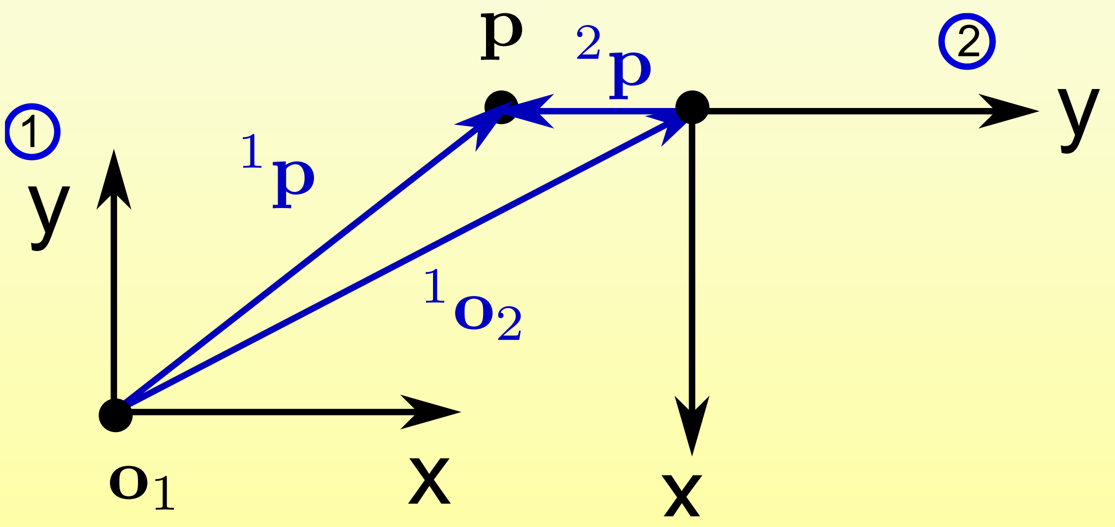
Conversion from rotation matrix to Euler angles

- $\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} c_\beta c_\gamma & s_\alpha s_\beta c_\gamma - c_\alpha s_\gamma & c_\alpha s_\beta c_\gamma + s_\alpha s_\gamma \\ c_\beta s_\gamma & s_\alpha s_\beta s_\gamma + c_\alpha c_\gamma & c_\alpha s_\beta s_\gamma - s_\alpha c_\gamma \\ -s_\beta & s_\alpha c_\beta & c_\alpha c_\beta \end{bmatrix}$
- The Euler angles can be derived as:
- $s_\beta = -r_{31}$
- There are two solutions for β:
  - $\beta = -\sin^{-1} r_{31}$
  - $\beta = 180° + \sin^{-1} r_{31}$
- We choose one solution, namely the solution satisfying $-90° \leq \beta \leq 90°$
  - Thus, we always have $\cos(\beta) \geq 0$
- $\tan \alpha = \frac{r_{32}}{r_{33}}$
- $\tan \gamma = \frac{r_{21}}{r_{11}}$

Uses of the homogeneous transformation matrix T

1. Description of frame relationship:

Given two frames, frame 2 and frame 1, the matrix **T** can be used to describe the full relation (orientation and translation) between the two frames.



2. Transforming the reference frame of a point **p** from frame 1 to frame 2:

$$ {}^1\tilde{\mathbf{p}}^1 = \mathbf{T}_2^1 \; {}^2\tilde{\mathbf{p}}^2 $$

Uses of the homogeneous transformation matrix $\mathbf{T}$

3. Transforming a point that moves with rigid-body motion.
   - A body moves with frame 2 attached to it.
   - Frame 1 is the initial / world reference frame.
   - A point $\mathbf{p_1}$ is transformed to the new position $\mathbf{p_2}$ during the rigid motion described by $\mathbf{T}_2^1$.
   - $\mathbf{p}_2 = \mathbf{T}_2^1 \mathbf{p}_1$
   - Note that $\mathbf{p_1}$ and $\mathbf{p_2}$ are homogeneous vectors, we dropped the ~ for simplicity.
   - $\mathbf{p}_1$ and $\mathbf{p}_2$ are referenced to frame 1