

University of Toronto Scarborough
MATA36
Calculus II for Physical Sciences
Text Book: Calculus Early Transcendentals 9th Edition by James
Stewart, Daniel Clegg, Saleem Watson

Partial fraction decomposition is the breaking down of a fraction into multiple fractions where the numerator is a constant. The multiple fractions, which are obtained by decomposing, if we add them, we get the original fraction. This helps in integrating the original fraction.

$$\int \frac{1}{(x-3)(x+5)} dx$$

First, we decompose the given fraction.

The first step is to factorize the denominator, here, the denominator is already factorized.

$$\frac{1}{(x-3)(x+5)}$$

$$\frac{A}{(x-3)} + \frac{B}{(x+5)}$$

A and B are two constants, the value of which we have to find out. To do that, multiply both sides of the equation (decomposed and undecomposed) by the factorized denominator.

$$\frac{1}{(x-3)(x+5)} (x-3)(x+5) = \left(\frac{A}{(x-3)} + \frac{B}{(x+5)} \right) (x-3)(x+5)$$

$$1 = A(x+5) + B(x-3)$$

Now, one strategy is to input certain values to x which will remove one of the constants. For example, if we input, x=3, then B(3-3) = 0, leaving only the constant A. Repeat the procedure for the other constant.

Input x = 3,

$$1 = A(3+5) + B(3-3)$$

$$1 = 8A$$

$$A = \frac{1}{8}$$