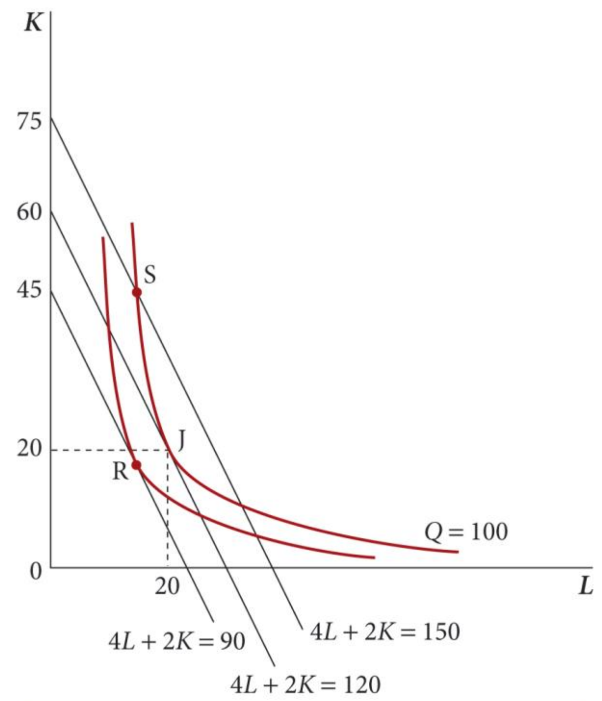
SOLUTION: COST MINIMISATION

- $V = 4L + 2K + \lambda\left(100 - 5K^{1/3}L^{2/3}\right)$
- $V = 4L + 2K + 100\lambda - 5\lambda K^{1/3}L^{2/3}$
- FOC: $\frac{\partial V}{\partial L} = \frac{\partial V}{\partial K} = \frac{\partial V}{\partial \lambda} = 0$
- $\frac{\partial V}{\partial L} = 4 - {}^{10}/_{3}\,\lambda K^{1/3}L^{-1/3} = 0 \Rightarrow 4 = {}^{10}/_{3}\,\lambda K^{1/3}L^{-1/3}$ ....(1)
- $\frac{\partial V}{\partial K} = 2 - {}^{5}/_{3}\,\lambda K^{-2/3}L^{2/3} = 0 \Rightarrow 2 = {}^{5}/_{3}\,\lambda K^{-2/3}L^{2/3}$ ....(2)
- $\frac{\partial V}{\partial \lambda} = 100 - 5K^{1/3}L^{2/3} = 0 \Rightarrow 100 = 5K^{1/3}L^{2/3}$ ....(3)
- Divide equation (1) by equation (2) – LHS of (1) by LHS of (2) and RHS of (1) by RHS of (2) $\rightarrow \frac{4}{2} = \frac{{}^{10}/_{3}\lambda K^{1/3}L^{-1/3}}{{}^{5}/_{3}\lambda K^{-2/3}L^{2/3}}$
- $\rightarrow 2 = 2K^{1/3-(-2/3)}L^{-1/3-2/3} \rightarrow 2 = 2KL^{-1};\ \therefore K = L$
- Substitute K (or L) into equation (3)
- $\rightarrow 100 = 5K^{1/3}K^{2/3} \Rightarrow 100 = 5K, \therefore K = 20 \Rightarrow L = 20$
- Solve for λ: Plug $K = L = 20$ into eq. (1) [or eq. (2)]: $4 = {}^{10}/_{3}\,\lambda(20)^{1/3}(20)^{-1/3}, \therefore \lambda = 1.2$
- This means that if production quota were increased by one unit, costs would increase by 1.2 (in this case, = MC



Given the input prices $w = 4$, $r = 2$, total cost of producing 100 units of output is minimized at J where the $Q = 100$ isoquant is tangent to an isocost line. A point such as R is inferior to J because although it is on a lower isocost line it is also on a lower isoquant. A point such as S is inferior to J because although it is on the same isoquant it is on a higher isocost line.

EXAMPLE 1: UTILITY MAXIMISATION

- A consumer's utility function is $U = x^{1/3}y^{2/3}$. The consumer has $R120$ to spend on x, y, while the price of $x = R4$ and the price of $y = R2$.

a. Find the values of x, y that maximize the consumer's utility, subject to the budget constraint, i.e. max $\left(x^{1/3}y^{2/3}\right)$ s.t. $120 = 4x + 2y$.

b. Show that $\frac{p_x}{p_y} = MRCS$ at the utility maximising levels of x, y