OUTLINE

1. Arithmetic and geometric series
2. Compounding
3. Discounting
4. Investment appraisal
5. Bonds and interest rates
6. Loan repayments
7. Annuities

THE IRR

- The IRR is the rate which yields the same return as the project after the same number of years
- Alternatively, the IRR is the rate which makes the PV of all payments zero
- Thus, the IRR is the rate of return required to increase the investment from a to y in x years
- *Rule*: invest in project if IRR exceeds market interest rate ($IRR > r$ - otherwise you could get a better return by just investing your money at the market interest rate)

SOLUTION: ANNUAL INSTALMENTS

- Annual instalment = $a = \frac{rK}{1-(1+r)^{-x}}$

$a = \frac{0.06(500000)}{1-(1+0.06)^{-25}} = 39113.35911.$

- Total value of instalments after 25 years:
- $xa = 25 \times 39113.35911 = 977833.9777$
- Principal = 500 000
- Therefore, total interest payments = $977833.9777 - 500000 = 477833.9777$
- Monthly instalment = $a = \frac{(^{r}/_{n})K}{1-(1+^{r}/_{n})^{-nx}}$

$a = \frac{(^{0.06}/_{12})500000}{1-(1+^{0.06}/_{12})^{-(25)(12)}} = 3221.5070.$

- Total value of instalments over 25 years = $nxa = 25 \times 12 \times 3221.5070 = 966452.1$
- Principal = 500 000
- ∴ total interest payments = $966452.1 - 500000 = 466452.1$

ANNUITY

- The future value of an annuity (with interest compounded annually) is:
- $FV = P_n = \frac{a}{r}[(1 + r)^x - 1]$
- The future value of an annuity (with interest compounded more than once a year) is:
- $FV = P_n = \frac{a}{r/n}[(1 + r/n)^{nx} - 1]$
- Useful to know this to determine how much the annuity will be worth after *x* years have expired; if we know the FV, we can determine the monthly deposit/payment required to reach our savings/investment goal

EXAMPLE 2

Suppose that a person takes out an annuity that is worth $R40\,000$ after 10 years, and that the interest rate is 7.5% p.a. How much must be deposited per month?

Solution:

- Monthly deposits, so interest period is 12
- Recall that

$FV = P_n = \frac{a}{r/n}[(1 + r/n)^{nx} - 1]$, where $P_n = R40\,000$, n=12, $x = 10$ and $r = 0.075$

- Plug in the values:

$$40000 = \frac{a}{0.075/12}\left[(1 + 0.075/12)^{(12)(10)} - 1\right].$$

- Now solve for a, the monthly deposit.
- Multiply both sides by $0.075/12$

$$40000 \times 0.075/12 = a\left[(1 + 0.075/12)^{(12)(10)} - 1\right].$$

- Divide both sides by $\left[(1 + 0.075/12)^{(12)(10)} - 1\right]$

$$\therefore a = \frac{40000(0.075/12)}{(1+0.075/12)^{(12)(10)}-1} = 224.81.$$