

CUBIC FUNCTIONS, Ch 5, P 153-60

- The cubic function $y = ax^3 + bx^2 + cx + d$
- The cubic function contains a term involving x^3 , x^2 and x and usually yields an S-shaped curve.
- If the cubic function cuts x axis three times, the corresponding cubic equation has 3 real roots.

See Fig 5.4

- If the cubic function cuts x axis once, so one real root (and two complex roots, involving imaginary numbers). See Fig 5.4
- If the cubic function has one cut plus a tangency, so 2 real roots with one of them "repeated".
- Unlike quadratic equations, there is no rule or formula for solving for the roots of cubic equations
- These roots can be solved only by trial-and-error methods.

EXAMPLE 2

- Suppose that Gross Domestic Product (GDP) grows continuously. Suppose that GDP needs to double in the next decade to halve the rates of unemployment and poverty.
 - What is the required GDP growth rate?
 - How many years would it take for GDP to double if the growth rate was 10% per year?

• Solution:

1. Since GDP grows continuously $y = ae^{rt}$, where $a = 1$; $y = 2$; $t = 10$

- $\therefore 2 = e^{10r} \Rightarrow \ln 2 = \ln e^{10r}$
- $\ln 2 = 10r \ln e$; $\therefore 10r = \ln 2$
- $\Rightarrow r = \frac{\ln 2}{10} = 0.0693 = 6.93\%$

2. Time taken to double with 10% growth rate

- $a = 1$; $y = 2$; $r = 10\% = 0.1$
- $\therefore 2 = e^{0.1t} \Rightarrow \ln 2 = \ln e^{0.1t} = 0.1t \ln e$
- $\therefore 0.1t = \ln 2 \Rightarrow t = 10 \ln 2 = 6.93 \approx 7$