ELASTICITY DEFINED

Preview from Notesale.co.uk
Page 4 of 18

- The elasticity of any function $y = f(x)$ is a way of measuring the relationship between a given proportionate change in x and the resulting proportionate change in the dependent variable y

- The elasticity shows how sensitive one variable (y) is to changes in another variable (x)

- In general, $E_{xy} = \dfrac{\text{percentage change in } y}{\text{percentage change in} x} = \dfrac{\%\Delta y}{\%\Delta x}$, where $\%\Delta y = \dfrac{\Delta y}{y_0} \times 100$, $\Delta x = \dfrac{\Delta x}{x_0} \times 100$

- Using differentiation: $E_{xy} = \dfrac{dy}{dx} \times \dfrac{x}{y} = \dfrac{dy/y}{dx/x}$

GENERALISED ELASTICITIES

Preview from Notesale.co.uk
Page 14 of 18

- A generalization: Suppose that $y = f(x)$
- Any elasticity can be obtained as $E^y = \frac{dy}{dx} \times \frac{x}{y}$
- Note that $\frac{dy}{dx}$ = marginal function of y; $\frac{y}{x}$ = average function of y
- Any elasticity can therefore be found as $E^y = \frac{\frac{dy}{dx}}{\frac{y}{x}} = \frac{marginal\ value\ of\ y}{average\ value\ of\ y}$

Example: total cost $TC = \mathrm{f}(q)$; Elasticity $E^{TC} = \frac{q}{TC}\frac{\mathrm{d}TC}{\mathrm{d}q}$

But $\frac{q}{TC}\frac{\mathrm{d}TC}{\mathrm{d}q}$ can be written as $\frac{\frac{\mathrm{d}TC}{\mathrm{d}q}}{\frac{TC}{q}} = \frac{MC}{AC}$

Thus, for any function, its elasticity is $\frac{\text{marginal value}}{\text{average value}}$

EXAMPLE 2

Preview from Notesale.co.uk
Page 17 of 18

- Suppose that demand is $q^d = p^{-2}e^{-2p}$. Find the price elasticity of demand (e^p) if $p = 0.5$.
- $E^p = \frac{dq^d}{dp} \times \frac{p}{q}$
- $[q^d]_{p=0.5} = 0.5^{-2} \times e^{-1} = 1.4715$
- Note: q^d is the product of two functions of p: $q^d = uv,\ u = p^{-2},\ v = e^{-2p}$. So, use product rule of differentiation to find $\frac{dq^d}{dp}$ …
- $\frac{dq^d}{dp} = u \cdot \frac{dv}{dp} + v \cdot \frac{du}{dp} = p^{-2}(-2e^{-2p}) + e^{-2p}(-2p^{-3}) = -2pe^{-2p} - 2p^{-3}e^{-2p} \ldots \left[\frac{dq^d}{dp}\right]_{p=0.5} = -8.8295$
- $\therefore [E^p]_{p=0.5, q^d=1.4715} = \frac{dq^d}{dp} \times \frac{p}{q} = -8.8295 \times \frac{0.5}{1.4715} = -3$
- Another simply and quick method is to substitute $p = 0.5$ at the final stage
- $E^p = \frac{dq^d}{dp} \times \frac{p}{q} = [p^{-2}(-2e^{-2p}) + e^{-2p}(-2p^{-3})] \times \frac{p}{q} = -2p^{-2}e^{-2p}(1 + p^{-1}) \times \frac{p}{p^{-2}e^{-2p}}$
- $\therefore [E^p]_{p=0.5} = -2\left(\frac{p+1}{p}\right) \times p = -2\,(0.5 + 1) = -3$
- Given that $|E^p| = 3 > 1$, the demand for this product is price elastic.