## **INVERSE OF 2 BY 2 MATRIX**

- Suppose that  $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ . Find the inverse of the matrix A as follows:
  - First, find Det(A). If  $Det(A) \neq 0$ , the matrix is non-singular and has an inverse
  - Now, find the inverse of A as

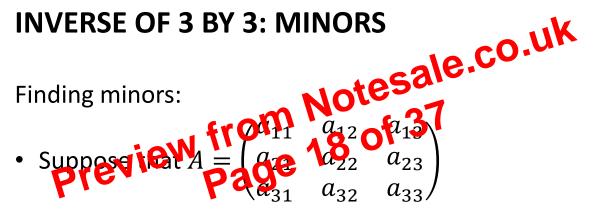
$$A^{-1} = \frac{1}{Det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \implies A^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$
$$A^{-1} = \begin{pmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}.$$

• Also do examples 19.6 & 19.7

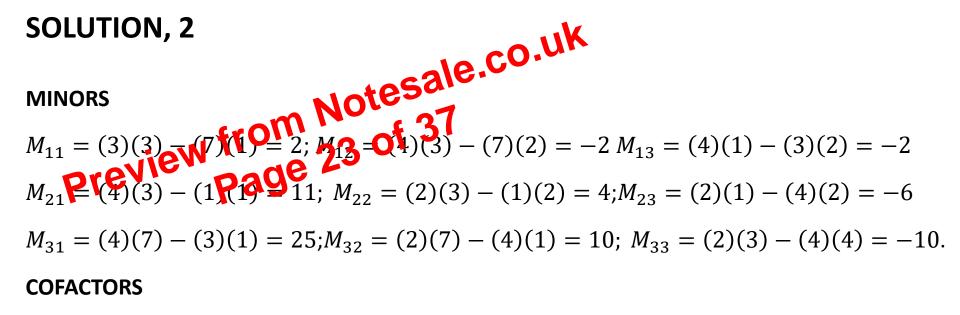
## INVERSE OF 3 BY 3 MATRIX

To find the inverse of a 3 by 3 matrix  $A^{-1} = \frac{1}{Det(A)} Adj(A)$ • First, find the marcolatopole of a 2 by 3 matrix  $A^{-1} = \frac{1}{Det(A)} Adj(A)$ 

- - New of a submatrix, found by deleting the row and column in which the
  - So the minor of element 1,1 is found by deleting row 1 and column 1, and finding the determinant of the resulting 2 by 2 submatrix
- Next, find the cofactors associated with each minor
  - A cofactor is merely a "signed" minor
- Next, find the cofactors associated with each minor
  - The sign is determined by  $-1^{i+j}$ , where i = row in which element is found and j = column in which element is found
  - Note that if i + j is even, then  $-1^{i+j} > 0$ , and sign is +; if i + j is odd, then  $-1^{i+j} < 0$ , and sign is -
- Next, find the adjoint matrix, Adj(A), which is just the transpose of the cofactor matrix, C
- The cofactor matrix is just a matrix containing all of the cofactors of the original matrix



- The minor for element  $a_{11}$  is found by deleting row 1 and column 1, and then finding the determinant of the resulting submatrix (2 by 2)
- This submatrix is  $M_{11} = \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix}$
- The determinant of this submatrix is  $|M_{11}|$ , which is the minor of element  $a_{11}$  $|M_{11}| = a_{22}a_{33} - a_{23}a_{32}$
- This means that the minor of  $a_{ij}$  is  $\left|M_{ij}\right|$
- See page 626-627



$$C_{11} = (-1)^2 |M_{11}| = 2; C_{12} = (-1)^3 |M_{12}| = 2; C_{13} = (-1)^4 |M_{13}| = -2$$
  

$$C_{21} = (-1)^3 |M_{21}| = -11; C_{22} = (-1)^4 |M_{22}| = 4; C_{23} = (-1)^5 |M_{23}| = 6.$$
  

$$C_{31} = (-1)^4 |M_{31}| = 25; C_{32} = (-1)^5 |M_{32}| = -10; C_{33} = (-1)^6 |M_{33}| = -10.$$
  
Now, write down the cofactor matrix:  $C = \begin{pmatrix} 2 & 2 & -2 \\ -11 & 4 & 6 \\ 25 & -10 & -10 \end{pmatrix}$ 

## • First, find Noteminant of A: $33 = \begin{pmatrix} 0.4 & 150 \\ 0.1 & -250 \end{pmatrix}$ t(A) = (0.4)(-250) - (150)(0.1)**SOLUTION - CRAMER'S RULE**

Find the determinants:

•  $Det(A_1) = (209)(-250) - (35)(150) = -57500$ 

• 
$$Det(A_2) = (0.4)(35) - (0.1)(209) = -6.9$$

Now, use Cramer's rule:

 $Y = \frac{Det(A_1)}{Det(A)} = \frac{-57500}{-115} = 500.$ 

• Now, find matrices  $A_1$  and  $A_2$  by replacing first and second column of A by  $b = \begin{pmatrix} 209 \\ 35 \end{pmatrix}$ 

$$r = \frac{Det(A_2)}{Det(A)} = \frac{-6.9}{-115} = 0.06.$$
$$x = \binom{500}{0.06}.$$

$$A_2 = \begin{pmatrix} 0.4 & 209 \\ 0.1 & 35 \end{pmatrix}$$

 $A_1 = \begin{pmatrix} 209 & 150 \\ 35 & -250 \end{pmatrix}.$ 

## **SOLUTION**

- Find the determinant of A:
- Notesale.co.uk • Note, A is 3 by 3 souse Sarrus fula.  $\begin{array}{c} \textbf{preview} & \textbf{and} & \textbf$

Det(A) = 2(3)(3) + 4(7)(2) + 1(4)(1) - 2(7)(1) - 4(4)(3) - 1(3)(2)

Det(A) = 10

Write down  $A_1, A_2, A_3$ :

2

1

• 
$$A_1 = \begin{pmatrix} 77 & 4 & 1 \\ 114 & 3 & 7 \\ 48 & 1 & 3 \end{pmatrix}; A_2 = \begin{pmatrix} 2 & 77 & 1 \\ 4 & 114 & 7 \\ 2 & 48 & 3 \end{pmatrix}; A_3 = \begin{pmatrix} 2 & 4 & 77 \\ 4 & 3 & 114 \\ 2 & 1 & 48 \end{pmatrix}$$

2

Now, find determinants (Sarrus' rule)