## INVERSE OF 2 BY 2 MATRIX



- Suppose that

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Find the inverse of the matrix $A$ as follows:

- First, find $Det(A)$. If $Det(A) \neq 0$, the matrix is non-singular and has an inverse
- Now, find the inverse of $A$ as

$$A^{-1} = \frac{1}{Det(A)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \Longrightarrow A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

$$A^{-1} = \begin{pmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{pmatrix}.$$

- Also do examples 19.6 & 19.7

## INVERSE OF 3 BY 3 MATRIX

To find the inverse of a 3 by 3 matrix A: $A^{-1} = \frac{1}{Det(A)} Adj(A)$

- First, find the minor of each element in the matrix
  - Minor = determinant of a submatrix, found by deleting the row and column in which the element is found
  - So the minor of element 1,1 is found by deleting row 1 and column 1, and finding the determinant of the resulting 2 by 2 submatrix
- Next, find the cofactors associated with each minor
  - A cofactor is merely a ”signed” minor
- Next, find the cofactors associated with each minor
  - The sign is determined by $-1^{i+j}$, where $i =$ row in which element is found and $j =$ column in which element is found
  - Note that if $i + j$ is even, then $-1^{i+j} > 0$, and sign is +; if $i + j$ is odd, then $-1^{i+j} < 0$, and sign is -
- Next, find the adjoint matrix, $Adj(A)$, which is just the transpose of the cofactor matrix, C
- The cofactor matrix is just a matrix containing all of the cofactors of the original matrix

# INVERSE OF 3 BY 3: MINORS



Finding minors:

- Suppose that $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$
- The minor for element $a_{11}$ is found by deleting row 1 and column 1, and then finding the determinant of the resulting submatrix (2 by 2)
- This submatrix is $M_{11} = \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix}$
- The determinant of this submatrix is $|M_{11}|$, which is the minor of element $a_{11}$
$$|M_{11}| = a_{22}a_{33} - a_{23}a_{32}$$
- This means that the minor of $a_{ij}$ is $|M_{ij}|$
- See page 626-627

# SOLUTION, 2



**MINORS**

$M_{11} = (3)(3) - (7)(1) = 2;\ M_{12} = (4)(3) - (7)(2) = -2\ M_{13} = (4)(1) - (3)(2) = -2$

$M_{21} = (4)(3) - (1)(1) = 11;\ M_{22} = (2)(3) - (1)(2) = 4; M_{23} = (2)(1) - (4)(2) = -6$

$M_{31} = (4)(7) - (3)(1) = 25; M_{32} = (2)(7) - (4)(1) = 10;\ M_{33} = (2)(3) - (4)(4) = -10.$

**COFACTORS**

$C_{11} = (-1)^2|M_{11}| = 2; C_{12} = (-1)^3|M_{12}| = 2; C_{13} = (-1)^4|M_{13}| = -2$

$C_{21} = (-1)^3|M_{21}| = -11; C_{22} = (-1)^4|M_{22}| = 4;\ C_{23} = (-1)^5|M_{23}| = 6.$

$C_{31} = (-1)^4|M_{31}| = 25;\ C_{32} = (-1)^5|M_{32}| = -10;\ C_{33} = (-1)^6|M_{33}| = -10.$

Now, write down the cofactor matrix: $C = \begin{pmatrix} 2 & 2 & -2 \\ -11 & 4 & 6 \\ 25 & -10 & -10 \end{pmatrix}$

# SOLUTION - CRAMER'S RULE



c) Cramer's rule:

- First, find determinant of $A$: $A = \begin{pmatrix} 0.4 & 150 \\ 0.1 & -250 \end{pmatrix}$

$Det(A) = (0.4)(-250) - (150)(0.1) = -115.$

- Now, find matrices $A_1$ and $A_2$ by replacing first and second column of $A$ by $b = \begin{pmatrix} 209 \\ 35 \end{pmatrix}$

$$A_1 = \begin{pmatrix} 209 & 150 \\ 35 & -250 \end{pmatrix}.$$

$$A_2 = \begin{pmatrix} 0.4 & 209 \\ 0.1 & 35 \end{pmatrix}.$$

Find the determinants:

- $Det(A_1) = (209)(-250) - (35)(150) = -57500$
- $Det(A_2) = (0.4)(35) - (0.1)(209) = -6.9$
- Now, use Cramer's rule:

$$Y = \frac{Det(A_1)}{Det(A)} = \frac{-57500}{-115} = 500.$$

$$r = \frac{Det(A_2)}{Det(A)} = \frac{-6.9}{-115} = 0.06.$$

$$x = \begin{pmatrix} 500 \\ 0.06 \end{pmatrix}.$$

## SOLUTION

- Find the determinant of A:

- Note, A is 3 by 3, so use Sarrus' rule ...

$$
\begin{array}{cccccc}
+ & + & + & - & - & - \\
2 & 4 & 1 & 2 & 4 & 1 \\
4 & 3 & 7 & 4 & 3 & 7 \\
2 & 1 & 3 & 2 & 1 & 3
\end{array}
$$

$$Det(A) = \begin{array}{cccccc} 2 & 4 & 1 & 2 & 4 & 1 \\ 4 & 3 & 7 & 4 & 3 & 7 \\ 2 & 1 & 3 & 2 & 1 & 3 \end{array}$$

$$Det(A) = 2(3)(3) + 4(7)(2) + 1(4)(1) - 2(7)(1) - 4(4)(3) - 1(3)(2)$$

$$Det(A) = 10$$

Write down $A_1, A_2, A_3$:

- $A_1 = \begin{pmatrix} 77 & 4 & 1 \\ 114 & 3 & 7 \\ 48 & 1 & 3 \end{pmatrix}; A_2 = \begin{pmatrix} 2 & 77 & 1 \\ 4 & 114 & 7 \\ 2 & 48 & 3 \end{pmatrix}; A_3 = \begin{pmatrix} 2 & 4 & 77 \\ 4 & 3 & 114 \\ 2 & 1 & 48 \end{pmatrix}$
- Now, find determinants (Sarrus' rule)