Now multiply (i) by x, ii) by y and then adding we get

$$\left(x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right) + \left(x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial y^2}\right) = n\left(x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}\right)$$

$$\because \frac{\partial^2 z}{\partial x\partial y} = \frac{\partial^2 z}{\partial y\partial x}$$

Now using (a) we get

$$nz + x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n^2 z$$

Hence,

$$x^2\left(\frac{\partial^2 z}{\partial x^2}\right) + 2xy\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n^2 z - nz = n(n-1)z$$

Example (i) $x^2 + 2xy + y^2$, degree $n = 2$

(ii) $2x^3 + xy^2 + z^3$, degree $n = 3$

(iii) $\dfrac{xy^2 - y^3}{2x + 3y}$, degree $n = 3 - 1 = 2$

Example: $f(x,y) = xy^3 + \dfrac{(x^2+y^2)^3}{2xy} + (x^2 - y^2)\sin 1/x$

$\rightarrow$ homo f^n with degree $n = 4$

then,

$$x^2 f_{xx} + 2xy f_{xy} + y^2 f_{yy} = 4(4-1)f(x,y)$$
$$= 2f(x,y)$$

Example