

SYLOW THEOREM

Sylow's Theorem and its Application

Def: Let G be a group & let P be a Prime A group of order p^α for some $\alpha \geq 1$ is called a P -group

A Subgroup of G is called P -Subgroup

* If G is a group of order $p^\alpha m$, where $p \nmid m$ then a subgroup of order p^α is called a Sylow P -subgroup of G .

* $Syl_p(G)$ = Set of Sylow P -subgroup of G
 $n_p(G)$ = No. of Sylow P -subgroup of G

Sylow's Theorems

(i) Sylow theorem - 1

Let G be a group of order $p^\alpha m$, where p is Prime not dividing m i.e. $p \nmid m$, then $\exists$ a Sylow P -Subgroup of G of order p^α i.e.

$$Syl_p(G) \neq \emptyset$$

OR

If $p^\alpha \mid |G|$ then $\exists H \leq G$ s.t. $|H| = p^\alpha$.

(ii) Sylow theorem - 2

Let P is a Sylow P -subgroup of G and Q is any P -subgroup of G . then $\exists g \in G$

s.t. $Q \leq gPg^{-1}$ i.e. is contained in some conjugate in G .

NB: In Particular, any two Sylow P -Subgroups are conjugate in G