B. REKHA

CALCULUS

PARTIAL DIFFERENTIATION

Partial Derivatives:

Let $z = f(x,y)$ be a f^n of two Variables x & y, Then Partial derivative of $z (= f)$ w.r.t x at Point (a,b) is defined as ___

$$z_x = f_x = \left.\frac{\partial f}{\partial x}\right|_{(a,b)} = \lim_{h \to 0} \frac{f(a+h, b) - f(a,b)}{h}$$

and

$$z_y = f_y = \left.\frac{\partial f}{\partial y}\right|_{(a,b)} = \left.\lim_{k \to 0} \frac{f(x, y+k) - f(x,y)}{k}\right|_{(a,b)}$$

$$= \lim_{k \to 0} \frac{f(a, b+k) - f(a,b)}{k}$$

Provided, the limit exists

(Note: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ are Called the first order Partial derivative)

Second order Partial derivative

(i) $\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$ ii) $\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$

iii) $\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial^2 z}{\partial y \partial x}$ iv) $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial^2 z}{\partial x \partial y}$

Note: Continuity of a f^n at a Point, doesn't imply that Partial derivative of that f^n at that Pt will exists