② Let $C = \cos\theta + \cos 3\theta + \cos 5\theta + \cdots \cos(2n-1)\theta$

Sol & $S = \sin\theta + \sin 3\theta + \sin 5\theta + \cdots \sin(2n-1)\theta$

$\Rightarrow C + iS = (\cos\theta + i\sin\theta) + (\cos 3\theta + i\sin 3\theta) + \cdots\cdots (\cos(2n-1)\theta + i\sin(2n-1)\theta)$

$= e^{i\theta} + e^{3i\theta} + e^{5i\theta} + \cdots e^{(2n-1)i\theta}$

Geometric Series

$a = e^{i\theta},\ n = n,\ r = e^{2i\theta}$

$S_n = \dfrac{a(r^n - 1)}{r-1}$

$\Rightarrow C + iS = \dfrac{e^{i\theta}\left(e^{2ni\theta} - 1\right)}{\left(e^{2i\theta} - 1\right)}$

$= \cancel{e^{i\theta}}\left[\dfrac{e^{ni\theta}\left(e^{ni\theta} - e^{-ni\theta}\right)}{\cancel{e^{i\theta}}\left(e^{i\theta} - e^{-i\theta}\right)}\right]$

$= \dfrac{e^{ni\theta}\left(\dfrac{e^{ni\theta} - e^{-ni\theta}}{2i}\right)}{\left(\dfrac{e^{i\theta} - e^{-i\theta}}{2i}\right)}$ $\div$ N & D by $2i$

$\Rightarrow C + iS = (\cos n\theta + i\sin n\theta)\left(\dfrac{\sin n\theta}{\sin\theta}\right)$

Comparing Real part only

$C = \cos n\theta \cdot \dfrac{\sin n\theta}{\sin\theta}$

$\cos\theta + \cos 3\theta + \cos 5\theta + \cdots \cos(2n-1)\theta = \cos n\theta \cdot \dfrac{\sin n\theta}{\sin\theta}$

$= \dfrac{2}{2}\cos n\theta\,\dfrac{\sin n\theta}{\sin\theta}$

$= \dfrac{\sin 2n\theta}{2\sin\theta}$

$\boxed{1.5-4}$

Q. $1 + x\cos\theta + x^2\cos 2\theta + \dots x^n\cos n\theta$

Sol: Let $C = 1 + x\cos\theta + x^2\cos 2\theta + \dots x^n\cos n\theta$

& $S = x\sin\theta + x^2\sin 2\theta + \dots x^n\sin n\theta$

$$C+iS = 1 + x(\cos\theta + i\sin\theta) + x^2(\cos 2\theta + i\sin 2\theta) + \dots x^n(\cos n\theta + i\sin n\theta)$$

$$= 1 + xe^{i\theta} + x^2 e^{i2\theta} + x^3 e^{i3\theta} + \dots + x^n e^{in\theta}$$

G. Series
$a = 1,\ r = xe^{i\theta}$
$n = n+1$
$S_n = \dfrac{a(r^n - 1)}{r-1}$

$$C+iS = 1\left[\frac{(xe^{i\theta})^{n+1} - 1}{xe^{i\theta} - 1}\right] = \frac{x^{n+1}e^{(n+1)i\theta} - 1}{xe^{i\theta} - 1}$$

$$= \frac{x^{n+1}\{\cos(n+1)\theta + i\sin(n+1)\theta\} - 1}{x(\cos\theta + i\sin\theta) - 1}$$

$$= \frac{(x^{n+1}\cos(n+1)\theta - 1) + i(x^{n+1}\sin(n+1)\theta)}{(x\cos\theta - 1) + i(x\sin\theta)}$$

$$= \frac{(x^{n+1}\cos(n+1)\theta - 1) + i(x^{n+1}\sin(n+1)\theta)}{(x\cos\theta - 1) + i(x\sin\theta)} \cdot \frac{(x\cos\theta - 1) - i(x\sin\theta)}{(x\cos\theta - 1) - i(x\sin\theta)}$$

$$C+iS = \frac{\{(x^{n+1}\cos(n+1)\theta - 1)(x\cos\theta - 1) + (x^{n+1}\sin(n+1)\theta)(x\sin\theta)\} + i\{x^{n+1}\sin(n+1)\theta(x\cos\theta - 1) - (x^{n+1}\cos(n+1)\theta - 1)(x\sin\theta)\}}{(x\cos\theta - 1)^2 + (x\sin\theta)^2}$$

Comparing real part only

$$C = \frac{x^{n+2}\cos(n+1)\theta\cos\theta - x^{n+1}\cos(n+1)\theta - x\cos\theta + 1 + x^{n+2}\sin(n+1)\theta\sin\theta}{x^2\cos^2\theta + 1 - 2x\cos\theta + x^2\sin^2\theta}$$

$$= \frac{x^{n+2}[\cos(n+1)\theta\cos\theta + \sin(n+1)\theta\sin\theta] - x^{n+1}\cos(n+1)\theta - x\cos\theta + 1}{x^2(\cos^2\theta + \sin^2\theta) - 2x\cos\theta + 1}$$

$$C = \frac{x^{n+2}\cos[(n+1)\theta - \theta] - x^{n+1}\cos(n+1)\theta - x\cos\theta + 1}{x^2 - 2x\cos\theta + 1}$$

$$C = \frac{x^{n+2}\cos n\theta - x^{n+1}\cos(n+1)\theta - x\cos\theta + 1}{x^2 - 2x\cos\theta + 1}$$ Ans