## **13. Inverse operations**

Suppose we pick a base, 2 say.

Suppose we pick a power, 8 say.

We will now raise the base 2 to the power 8, to give  $2^8$ .

Suppose we now take logarithms to base 2 of  $2^8$ .

We then have

 $\log_2 2^8$ 

Using the laws of logarithms we can write this as

 $8\log_2 2$ 

Recall that  $\log_a a = 1$ , so  $\log_2 2 = 1$ , and so we have simply 8 again, the number we started with.

So, raising the base 2 to a power, and then finding the logarithm to base 2 of the result are inverse operations.

Let's look at this another way.

Suppose we pick a number, 8 say.

Suppose we find its logarithm to base 2, to evaluate  $\log_2 8$ .

Suppose we now raise the base 2 to this power:  $2^{\log_2 8}$ .

Because  $8 = 2^3$  we can write this as  $2^{\log_2 2^3}$ . Using the laws of coarithms this equals  $2^{3\log_2 2}$  which equals  $2^3$  or 8, since  $\log_2 2 = 1$ . We see that a single the base 2 to the logarithm of a number to base 2 results in the original number

So raising a base to a power, and finding the logarithm to that base are inverse operations. Doing one operation, and the explosing it by the other we end up where we started.

## Example

Suppose we are working in base e. We can pick a number x and evaluate  $e^x$ . If we follow this by taking logarithms to base e we obtain

 $\ln e^x$ 

Using the laws of logarithms this equals

 $x \ln e$ 

but  $\ln e = 1$  and so we are left with simply x again. So, raising the base e to a power, and then finding logarithms to base e are inverse operations.

## Example

Suppose we are working in base 10. We can pick a number x and evaluate  $10^x$ . If we follow this by taking logarithms to base 10 we obtain

 $\log 10^x$ 

Using the laws of logarithms this equals

## $x \log 10$

but  $\log 10 = 1$  and so we are left with simply x again. So, raising the base 10 to a power, and then finding logarithms to base 10 are inverse operations.

10

